INVESTIGATING CHARACTERISTICS OF MALAWIAN PRIMARY SCHOOL TEACHERS' MATHEMATICAL KNOWLEDGE FOR TEACHING EARLY ALGEBRAIC CONCEPTS: EQUALS SIGN, PATTERNS, VARIABLES AND LINEAR EQUATIONS

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GETRUDE DEBORAH ZIMBA JUMBE

UNIVERSITY OF MALAWI
CHANCELLOR COLLEGE

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By

Getrude Deborah Zimba Jumbe
Bachelor of Education (Science) –University of Malawi

Thesis submitted in partial fulfilment of the requirements for the degree of Master of Education (Curriculum Studies in Mathematics and Science Education)

University of Malawi
Chancellor College

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DECLARATION

I the undersigned hereby declare that this thesis/dissertation is my own original work which has not been submitted to any other institution for similar purposes. Where other people's work has been used acknowledgements have been made.

Getrude Deborah Zimba Jumbe

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Signature

_____

Date

CERTIFICATE OF APPROVAL

The undersigned certify that this thesis represents the learner's own work and effort and has been submitted with our approval.

Signature : _____ Date : _____

Arne Jakobsen, PhD (Professor)

Main Supervisor

Signature: _____Date: _____

Levis Eneya, (PhD)

Co-Supervisor

DEDICATION

I dedicate this work to my mum, Staphel Chimaliro Zimba and late father, Benson Charles Zimba whose love and pieces of advice have taken me this far. I also dedicate this work to our son Ekari and daughter Mahala, they are such understanding kids. May God bless them.

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ABSTRACT

The purpose of this study was to investigate characteristics of Malawian primary school teachers' mathematical knowledge for teaching (MKT) early algebraic concepts of equals sign, patterns, variable and linear equations. Further, the study examined if school location (urban or rural) and teachers' experience affected teachers' MKT of the algebraic concepts. The sample comprised 35 teachers from 2 urban and 3 rural schools. The study employed mixed methods research design. Quantitative data were generated inform of scores of a multiple choice test which was administered to 35 teachers in the 5 schools. The test items were adapted from the Learning Mathematics for Teaching Project, and Measures of Effective Teaching Projects at the University of Michigan in the United States of America. One item was adapted from Mamba (2011) test items. In addition, each teacher filled demographic data form. After the test, at each school, the teachers participated in focus group discussions. Quantitative data were analysed using Microsoft Excel while qualitative data were analysed thematically. The findings revealed that teachers did not have adequate MKT of the algebraic concepts of equals sign, patterns, variable and linear equations. Further, the study revealed that school location did not affect teachers' MKT of early algebraic concepts. The study revealed that more experienced had more MKT of early algebraic concepts than inexperienced teachers.

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LIST OF ACRONYMNS

CCK:	Common Content Knowledge
FGD:	Focus Group Discussion
HCK:	Horizon Content Knowledge
KCL:	Knowledge of Content and Learners
KCT:	Knowledge of Content and Teaching
MKT:	Mathematical Knowledge for Teaching
MSCE:	Malawi School Certificate of Education
NORHED:	Norwegian Programme for Capacity Building in Higher Education and Research for Development
SACMEQ:	Southern and Eastern Africa Consortium for Monitoring Educational Quality
TTC:	Teacher Training College

CHAPTER 1

INTRODUCTION

This chapter presents background information of the study, problem statement, purpose of the study, research questions, significance and limitations of the research. The chapter ends with a summary.

1.1 Background information

Mathematical knowledge for teaching can be defined as the mathematical knowledge used to carry out the work of teaching mathematics (Hill, Rowman, & Ball, 2005). Examples of mathematical work of teaching include presenting mathematical ideas, responding to learners' "why" questions, recognizing what is involved in using a particular representation, connecting a topic being taught to topics from previous or future years, modifying tasks to be either easier or harder to cater for learners' diversity, giving or evaluating mathematical explanations, choosing and developing useable definitions, and asking productive mathematical questions (Ball, Thames & Phelps, 2008)

Different studies have been conducted in the area of mathematical knowledge for teaching (MKT). Hill et al. (2005) conducted a study in United States of America. The study explored whether and how teachers' mathematical knowledge for teaching contributes to gains in learners' mathematical achievements. The study revealed that teachers' MKT was significantly related to learners' achievement. The authors

concluded that learners' success in mathematics can be achieved by improving teachers' knowledge.

Further, Turnuklu and Yesildere (2007) conducted a study in Turkey. The aim of their study was to determine the pre-service primary mathematics teachers' competency in pedagogical content knowledge in mathematics. The study revealed that deep understanding of subject matter was necessary but not adequate to teach maths. The study pointed out connection between content knowledge and knowledge for teaching mathematics. The conclusion of the study was that primary school mathematics teachers must be educated on both; mathematics knowledge and pedagogical content knowledge aspects.

Furthermore, Gencturk (2012) in his doctoral thesis in the United States of America examined the relationship among teachers' mathematical knowledge and their practices and learner achievement. The sample frame of the study comprised 21 teachers who enrolled in a master's programme designed specifically for the needs of the partnership. The teachers were followed for four years to study how their mathematical knowledge changed over time. The results indicated that teachers' MKT increased radically and the teachers made statistically significant changes in their lessons design and delivery.

Similarly, Jakobsen and Kazima (2015) conducted a study on MKT among primary school learner teachers in Malawi. Mathematics teacher educators from different teacher training colleges participated in the study. The participants answered a questionnaire about different aspects of learner teachers' MKT with a special focus on

subject matter knowledge. The study revealed that what learner teachers struggle least with, at teacher training college was reported to be most difficult for the learner teachers to teach at primary schools. However, in the study data were collected from teacher educators only. The authors acknowledged that there was need to collect data from the learner teacher themselves as well.

Similarly, Malawi Institute of Education (2014) conducted a study aimed at assessing teachers' pedagogical content knowledge in English, Chichewa and Mathematics in standards 5, 6 and 7 of primary schools in Malawi. One of the objectives of the study was to find out the level of pedagogical content knowledge of teachers in English, Chichewa and Mathematics in standards 5 to 7. The study revealed that most mathematics teachers had poor pedagogical content knowledge and subject matter knowledge in the subjects.

This study investigated characteristics of Malawian primary school mathematics teachers' mathematical knowledge for teaching early algebraic concepts of equals sign, variable, patterns and linear equations. Further, the study investigated the effects of school location (urban or rural school) and teaching experience on teachers' MKT of early algebraic concepts.

1.2 Statement of the problem

According to the Southern and Eastern Africa Consortium for Monitoring Educational Quality (SACMEQ), (2010); Malawi National Examinations Board (2014); Malawi National Examinations Board (2015), in Malawi, learners perform poorly in primary school mathematics. Studies and reports reveal several factors which contribute to this

poor performance in primary school mathematics. The factors include large classes which hinder teacher-learner interaction, inadequate teaching and learning resources, teachers' limited knowledge of learner centred approaches and inadequate qualified teachers (International Mathematical Union, 2014; National Education Sector Plan, 2008). Studies conducted after the interventions, still revealed that primary school learners perform poorly in mathematics (SACMEQ, 2010; Malawi Institute of Education, 2014).

The primary school curriculum was reviewed in 2006. Use of learner centred approaches is the primary focus of the reviewed primary school curriculum (Malawi Institute of Education, 2006). Currently, pre-service and in-service teachers are trained on how to teach using learner centred approaches. Teaching using learner centred approaches require teachers to have adequate knowledge of the subject matter and pedagogical knowledge in order to carry out the work of teaching effectively (Stuart, Akyeampong & Croft, 2009).

Pre-algebra and algebraic concepts form a big percentage of the content of primary school mathematics (Malawi Institute of Education, 2005; Malawi Institute of Education,2006; Malawi Institute of Education, 2007). In addition, primary school algebra forms a basis for secondary school algebra. Despite the review of primary school curriculum where use of learner centred approaches is encouraged, Malawi National Examinations Board (2014); Malawi National Examinations Board (2015); Malawi National Examinations Board (2016) indicate that most learners perform poorly in algebraic concepts like patterns and equations. In addition, generally learners from urban schools perform better than learners from rural schools.

Furthermore, Ball et al. (2008) argue that teaching experience is one factor which affects teachers' MKT. The problem, therefore, is in Malawi, no study has been conducted to examine characteristics of teachers' MKT of early algebraic concepts of equals sign, variable, patterns and linear equations and explore if factors such a school location and teaching experience affect teachers' MKT about the concepts, hence this study.

1.3 Purpose of the study

The purpose of this study was to investigate characteristics of primary school teachers' MKT about the early algebraic concepts of equal signs, patterns, variable and linear equations and explore if school location and teachers' teaching experience affected their MKT of the algebraic concepts.

1.4 Research questions

Main research question

What is the MKT for teaching Algebraic concepts in Malawian primary school?

Supporting research questions;

1. What characterise Malawian primary school teachers' MKT of early algebraic concepts?
2. Are there differences in MKT of early algebraic concepts between teachers of urban and rural schools?
3. Are there differences in MKT of early algebraic concepts between groups of experienced and inexperienced teachers?

1.5 Significance of the study

The study contributes to the body of knowledge regarding characteristics of teachers' MKT of early algebraic concepts in Malawi. It is hoped that results from this study will inform various stakeholders like curriculum developers, and continuous professional development designers about content to include in the mathematics curriculum. This will help equip both pre-service and in-service teachers with adequate and relevant MKT of early algebraic concepts. In addition, primary teacher trainers may be guided by this study on what and how to teach early algebraic concepts to pre-service teachers so that they become effective mathematics teachers. To Malawi National Examination Board- the board in Malawi which currently certifies teachers, the study may give insights into what and how to assess and certify teachers.

1.6 Limitations of the research

This study was trying to examine characteristics of teachers' MKT of early algebraic concepts. However, the focus was on two domains of MKT-common content knowledge and knowledge of content and learners. Other domains of MKT were less discussed due to limited time. Another limitation is that the study was conducted to five schools only due to financial and time factor. Therefore, the findings may be generalised to schools of similar characteristics only.

1.7 Chapter summary

This chapter outlined background to the study. In addition, problem and purpose of the study have been stated. The study is important because it will inform stakeholders about characteristics of teachers' MKT of early algebraic concepts. The chapter ends

by informing some of the limitations of the study. In the following chapter, I will discuss literature review in relation to the study.

CHAPTER 2

LITERATURE REVIEW

This chapter reviews relevant literature about history of algebra, algebraic concepts for primary schools and how they are taught in different regions of the world. Furthermore, this chapter analyses literature on common errors and misconceptions learners have on the algebra. Besides, the chapter provides a theoretical framework of the study. It ends with a chapter summary.

2.1The concept of Algebra

Haylock (2010) defines algebra as a branch of mathematics where letters are used to represent variables in order to express generalisations. Maclaurin (1748) as cited in Katz (2007) defines algebra as a general method of computation by certain signs and symbols which have been designed for this purpose, and found convenient. Euler (1984) as cited in Katz (2007) defines algebra as science which teaches how to determine unknown quantities by means of those that are known. However, when most learners are asked what algebra is, they often say it is arithmetic with letters (Suggate, Davis and Goulding, 2006)

Algebra is one of the oldest branches of mathematics which has evolved over years. Katz (2007) argues that historical development of algebra is in three stages: the rhetorical stage, the syncopated stage, and the symbolic stage. Rhetorical is the stage

where all statements and arguments are made in words and sentences. Syncopated is the stage where some abbreviations are used when dealing with algebraic expressions. In the symbolic stage, there is total symbolization that is all numbers, operations and relationships are expressed through a set of well-known symbols and rules.

Besides these three stages of expressing algebraic ideas, Katz (2007) further argues that there are four conceptual stages that have happened. These are firstly, the geometric stage, where most of the concepts of algebra are geometric. Secondly, the static equation-solving stage, where the goal is to find numbers satisfying certain relationships. Thirdly, the dynamic function stage, this is where motion seems to be an underlying idea. Finally, the abstract stage, here structure is the goal. Naturally, neither these stages nor the earlier three are disjoint from one another; there is always some overlap.

The nineteenth century marks the beginning of modern algebra. Modern algebra is used in solving equations. In addition, modern algebra provides the language and patterns of reasoning used in other branches of mathematics like geometry. The word "algebra" comes from a book called *Al-jabr w'al mˆuqabala*by Al-Kwarizmi the Arabic mathematician. It was a blend of many mathematical traditions including Babylonian, Indian and Greek and included the problems of inheritance and property division. Al-Kwarizmi's algebra went no further than the solution of quadratic equations (Stillwell, 2010).

2.2 Algebra in primary schools

Most primary school mathematics curricula separate the study of arithmetic and algebra. Arithmetic is mainly taught in early years of primary school, and algebra in later years. Cai and Knuth (2011) state that there is a growing consensus that this separation makes it more difficult for learners to learn algebra in the later years. Further, based on recent research on learning, there are many obvious and widely accepted reasons for developing algebraic ideas in the earlier years.

Curriculum developers, educational researchers, and policy makers have started exploring the kinds of mathematical experiences primary school learners need to prepare them for the formal study of algebra at the later years. For example, there is evidence that United States learners are ill-prepared for the study of algebra (Silver and Kenney, 2001). According to a rigorous academic analysis by the American Association for the Advancement of Sciences, the majority of textbooks used for algebra in the United States have serious weaknesses. In addition, there is evidence that most elementary school teachers in the United States are not adequately prepared to integrate algebraic reasoning into their instructional practices. Most United States learners do not start the formal study of algebra until eighth or ninth grade, and many of them experience difficulties making the transition from arithmetic to algebra because they have little or no prior experience with the subject.

According to Cai and Knuth (2011) in several countries such as China and Singapore, learners begin the formal study of algebra much earlier. In China, the main algebra-related goal in the Chinese elementary curriculum is for learners to better represent and understand quantitative relationships, numerically and symbolically. The major

focus is on equations and equation solving. Variables, equations, equation solving, and function sense saturate the curriculum in grades 1 to 4. Equations and equation solving are formally introduced in the first half of grade 5. Once equation solving is introduced, it is applied to the learning of mathematical topics, such as fractions, statistics, and proportional reasoning, in the second half of grade 5 and 6. The term 'variable' is not formally defined in Chinese elementary school mathematics. However, in the teacher's guide for the national curriculum, teachers are reminded that variables can represent many numbers simultaneously, that they haveno place value, and that representations of variables can be selected subjectively.

Malawi follows 8-4-4 education system, where learners spend 8 years at primary school, 4 years at secondary school and 4 years for tertiary education. The current Malawian primary school mathematics curriculum has six core elements. These are firstly, number concepts and operations; secondly, patterns, functions and algebra; thirdly, data handling; fourthly, accounting and business studies; fifthly, measurement; and lastly, space and shapes (Malawi Institute of Education, 2007; Malawi Institute of Education, 2008). In all the core elements there are elements of algebra. For example, the core element of number concepts and operations involve computation of basic operations where equals sign is frequently used. The core element of patterns, functions and algebra caters for topics such as linear equations, like and unlike terms. Data handling is about graphs. In measurement learners measure and calculate measurements of different objects, in order to achieve this, they need knowledge of equality and equations.

Pre-algebraic concepts are introduced to learners right away in standard one; for example, standard 1 children learn patterns and picture graphs (Ministry of education, 2005). From standard 5 learners are introduced to symbolic algebra (Ministry of Education,2007). They learn concepts such as variables, number patterns, linear equations and inequalities. Figure 2.1 summarises algebraic concepts taught in Malawian primary school mathematics.

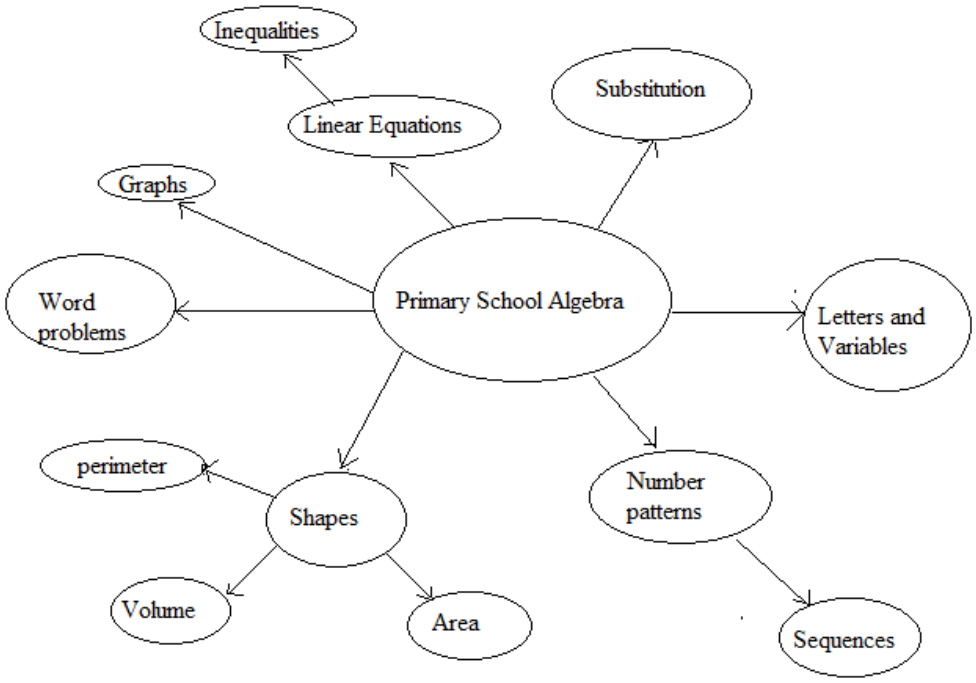

Figure 2.1: Concept map of algebraic concepts taught in Malawian primary schools

Source: Developed by researcher

2.3 Learners' misconceptions of algebraic concepts

Hansen (2011) highlighted that errors and misconceptions are inevitable when teaching mathematics and algebra in particular. She argues that errors could be made

for many reasons including carelessness, misinterpretation of symbols, lack of relevant knowledge, inability to check the answer given, or the result of misconception. The term 'misconception' is used when a learner's conception is considered to be in conflict with the accepted meanings and understandings in mathematics. Misconception could be interpretation of a rule, an over or under generalization of a rule, or an alternative conception of the situation. Ryan and Williams (2007) as cited in Hansen (2011) note that misconceptions are more challenging than errors because they are set within deeper levels of knowledge, therefore they demand diagnosis and dialogue to ascertain them, and this demand teacher to have adequate MKT (Ball et al., 2008).

Cockcroft (1982) as cited in Orton &Frobisher (2005) claim that algebra is a source of confusion and negative attitudes among learners. They suggested two main reasons these are unhelpful teaching and learning difficulties. Orton and Frobisher (2005) propose that great care should be taken when teaching algebra.

Studies have shown that most learners develop several misconceptions of algebraic concepts. For example, Suggate et al. (2006) point out that one of the challenges when solving an algebraic problem is to present the answer one to an equation. Haylock (2010) argues that learners all over the world develop almost the same misconceptions about algebraic concepts. According to Haylock (2010) amongst common algebraic concepts which leaners develop are: equal sign, variables and equations. Haylock (2010) explains that equals sign in algebra means equivalence. However, since in arithmetic the equal sign mean 'as a result of' or an indication to carry an operation, many learners continue with such perception.

Secondly, learners also have misconceptions with the concept of variable: In mathematics, letters are used in different ways. For example, are used to label figures and are abbreviations for units (for example, g for grams). Therefore, some learners have problems when in algebra the same letter is introduced as a variable or a parameter (Haylock, 2010). Thirdly**,** in the concept of linear equations, studies show that many learners view the equals sign as an instruction to do something, rather than an indication of a relationship (Orton and Frobisher, 2005; Haylock,2010)

Bromme and Steinbring (1994) argue that learners are better able to understand and remember concepts and principles if they have meaning for them. There is therefore the need to analyse the PCK of teachers with respect to the development of meaning for their learners. In order that effective learning may take place, the teacher has to help learners have proper conceptual understanding, identify the learners' misconceptions in the subject and employ appropriate teaching techniques to reduce the misconceptions to the minimum.

2.4 Primary school teacher education in Malawi

In Malawi, primary school teachers are trained at teacher training colleges (TTC). Currently, there are eight public TTCs and some private TTCs. The learner teachers in both public and private TTCs follow the same curricula. There have been different primary teacher education programmes since 1964 when Malawi got her independence. However, what is common with all teacher training programmes is that the training duration is two years.

Since 2006, primary school teachers are trained under a programme called *Initial Primary Teacher Education.* The entry academic requirements into TTC include a Malawi School Certificate of Education (MSCE) with a pass in mathematics while sometimes a pass in Malawi School Certificate of Education mathematics is not a requirement. In Malawi, a person is awarded a Malawi School Certificate of Education after successfully studying for eight years at primary school and four years at secondary school (Ministry of Education, 2007). After graduation, teachers are supposed to teach in rural area schools for 5 years before posted to urban area schools. This is the case because most rural schools have few teachers and most teachers do not like working in rural areas.

In Malawi, just like many countries, in the course of teaching teachers attend in-service trainings. Sometimes the trainings are organised by schools or Ministry of Education while in other cases by non-governmental organisations.

It is common in Malawi, for qualified teachers to privately pursue MSCE studies to improve grades. In most cases, the teachers pursue the studies at Open Secondary Schools. Open Secondary Schools offer secondary education; the lessons are offered in the afternoon or evenings. The schools mainly target adults who withdrew from secondary education or want to improve MSCE grades.

2.6 Previous studies on teachers' MKT about early algebraic concepts

There have been several studies on MKT of elementary algebra. Asquith, Stephens, Knuth and Alibali (2007) did a study in United States of America where theyfocused on teachers' knowledge of learners' understanding of core algebraic concepts of equals sign and variable. They found that teachers did not have adequate knowledge

of learners' thinking; however, they only focused on the concepts equals sign and variable.

Osei (2000) in his doctorate study in South Africa examined pre-service teachers' knowledge and understanding of algebraic concepts of South African primary pre-service teachers. He found that pre-service teachers did not have adequate content knowledge in algebra. However, he focused on pre-service teachers who did not have adequate teaching experience. Ball et al. (2008) argue that teaching experience is one of the factors which affect teacher's MKT in such a way that the more the teaching experience a teacher has the more the teacher's MKT.

Haylock (2010), who worked in teacher education for more than 30 years in Europe and some African countries like Kenya and Lesotho, noted that there are several misconceptions which learners make when they are learning concepts of early algebra such as equal signs, variable and equations. For example, learners may think equals sign always mean the same thing. From her experience, she observed that these misconceptions are almost the same to all learners regardless of their cultural contexts. However, her focus was on learners. She did not investigate if teachers had adequate teacher MKT to clear the misconceptions.

In Malawi, Mamba (2011) did a study on secondary school learners' misconceptions in linear algebra. She found that the learners had misconceptions. However, her focus was secondary school learners. Besides, Malawi Institute of Education (2014) conducted research to investigate PCK of primary school teachers. However, the study focused on general PCK in mathematics, English and Chichewa. The focus was

not in algebra. Therefore, this study focused on the extent to which primary school teachers have MKT to teach early algebraic concepts, as well as factors which affect the mathematical knowledge for teaching.

2.7 The MKT theoretical framework

Teachers are one of the key components of the reform of teaching and learning mathematics. Various structures of knowledge that teachers possess reveal what kind of teachers they might be. This fact raises the item "What should teachers know?" The answer can be explained by the concept of pedagogical content knowledge (PCK). Shulman (1986) proposed the concept of PCK that has a different meaning from content knowledge, which obviously must be possessed by teachers. Shulman stated that pedagogical content knowledge is a specific mixture of subject matter knowledge and pedagogical knowledge.

The pedagogical content knowledge is a kind of knowledge that reveals a teacher's meaningful and effective ways of teaching. In other words, it is a special kind of information generated from the transformation of the subject-matter knowledge that teachers have and it consists of several components namely: subject matter knowledge, pedagogical content knowledge, general pedagogical knowledge, and knowledge of learners, knowledge of curriculum, knowledge of educational contexts and knowledge of educational purposes. After Shulman's study, more researchers focused on the pedagogical content knowledge and demonstrated these components in different ways.

This study was guided by Ball et al. (2008) theory of MKT. Their work was an extension of Shulman's conceptualization of teacher knowledge for mathematics teachers. Ball et al. (2008) categorise MKT into six domains. (See figure 2.2 below).

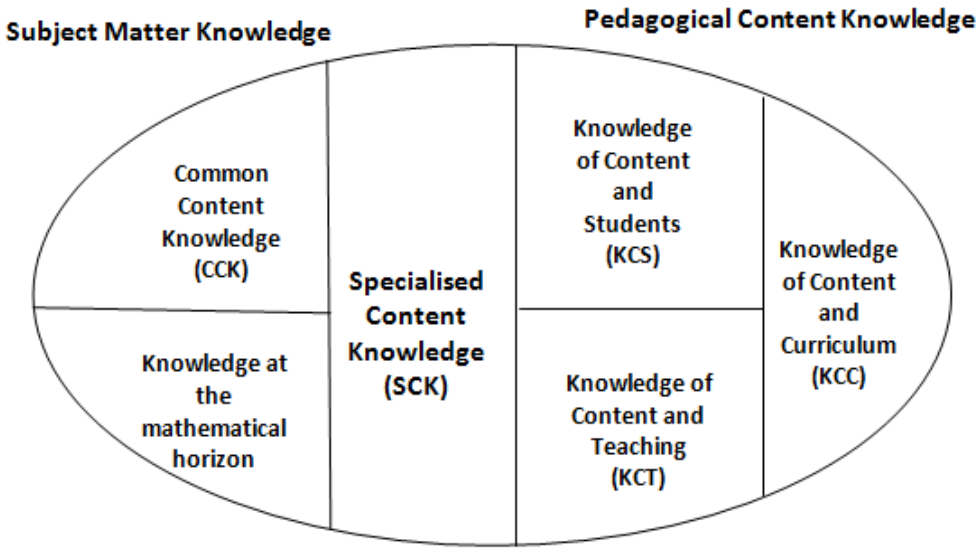

Figure 2.2: Mathematical knowledge for teaching (Ball et al, 2008)

Ball et al. (2008) define the six domains of teacher knowledge as follows:

Firstly, Common Content Knowledge(CCK), which is knowledge of mathematics that allows one to successfully solve mathematical problems other than for teaching however, it is not unique for the profession of teaching. Secondly, Specialised Content Knowledge (SCK), which is knowledge of how and why mathematical procedures and rules work, along with the knowledge of structuring and representing mathematical content. This knowledge includes the ability to appraise learners' ideas and computations and analyse the mathematical validity of unconventional learner methods. Thirdly, Horizon Content Knowledge (HCK), which is knowledge which helps teachers articulate how the mathematics they teach, fits into the mathematics

which comes later. Fourthly, Knowledge of Content and Learners this is knowledge of how learners analyse and comprehend mathematical ideas. This knowledge entails an understanding of common learner conceptions, misconceptions difficulties, errors, and interests in the field of mathematics. The fifth domain of teacher knowledge is Knowledge of Content and Teaching (KCT), which is knowledge of how to select and implement mathematically specific pedagogical strategies (e.g. analogies, illustrations, examples, explanations, demonstrations) to best explain and/or represent mathematical concepts in various instructional situations. The last domain of teacher knowledge according to Ball et al. (2008) is Knowledge of content and curriculum, which is knowledge of how the mathematical concepts that learners are learning relate to the various topics being concurrently addressed in other courses, as well as a holistic awareness of the appropriate mathematics curriculum.

Ball et al. (2008) pointed out that much as the MKT is divided into 6 domains, sometimes the boundaries are not clear. In this study I mainly focussed much on two domains these are Common Content Knowledge and Specialised Content Knowledge.

2.8 Chapter summary

This chapter discussed history of algebra, in particular the concept of algebra, highlighted main algebraic concepts in Malawian primary school mathematics curriculum and further reviewed international, regional and national literature on MKT of algebraic concepts. Theoretical framework which guided the study has also been deliberated. The chapter ends with a summary. In the following chapter I will discuss methodology employed in this study.

CHAPTER 3

METHODOLOGY

This chapter discusses the research design, sampling procedure, data collection instruments, data collection procedures and data analysis. The issues of ethical consideration are also discussed under data collection procedures. The chapter ends with a brief summary.

3.1 Research design

Research designs are plans and actions for research that include detailed methods of data collection and analysis. Creswell (2009) points out that there are several factors that guide the research design. These include; nature of research problem, personal experiences of a researcher, audience the research is writing for, and the research sample. The overall decision of research design is guided by the worldview assumptions the researcher brings to the study; procedures of inquiry; and specific methods of data collection, as well as data analysis (Creswell, 2009).

This study employed mixed methods approach. Mixed methods approach is where the researcher follows both the principles and assumptions of qualitative and quantitative study. In particular, this study followed sequential mixed method approach. In this approach, the researcher expands on the findings of one method with another (Creswell, 2009).

In the study, a test was used to collect quantitative data. The test scores were examined against work experience and school setting of the participants who in this study were primary school teachers.

Creswell (2009) further defines qualitative design as a means of understanding the meaning individuals or groups give to a problem. The study used focus group discussion method to understand the responses which teachers gave to the test. Thus, the aim of using both quantitative and qualitative methods was to gain deep understanding of the context. O'Leary (2010) argues that mixed methods approach assist in adding scope and depth to a study. Furthermore, the approach help in ensuring validity and reliability as triangulation nullifies biasness that would occur if one method is used.

3.2 Sampling procedure

Fraenkel and Wallen (2000); Cohen, Manion and Morrison (2007) define sample as any group from which a researcher gets information, and refers to sampling as the process of selecting the individuals. In this study, I sampled five primary schools of a district in the Southern Region of Malawi; two were urban schools while three were rural. Both urban and rural schools were sampled because I wanted to have a fair representation in terms of the school context.

Four of the sampled schools were teaching practice schools for the institution of the researcher while the other school was close to the researcher's workplace. I employed purposive sampling to choose teachers who participated in the study. That is only standards 5 to 8 mathematics teachers who did not have a class during the time of my

visit participated in the study. This was so to avoid disturbing lessons. At each school I managed to collect data from 6 to 12 teachers. In total 35 teachers participated in the study;18were teachers from rural schools and 17 from urban schools. Table 3.1 summarises characteristics of the sample

Table 3.1: Characteristics of the sample

		Total
No. of teachers by gender	Males	14
	Females	21
No. of teachers by school location	Urban	17
	Rural	18
No. of teachers by teaching experience	0 to 2 years	9
	3 to 10 years	11
	11 years and above	15

3.3 Characteristics of the schools during the time of data collection

(i) Primary School A

This was an urban school. It was a demonstration school for a teacher training colleges. All teachers at this school were qualified. Normally teachers who performed well and had served for at least five years were posted to this school. Learner teachers from a nearby Teacher Training College used this school for teaching practice. The school had inadequate teaching and learning materials at the time of the visit.

However, some teachers as well as learners from this school could borrow books from the teacher training college library.

The average number of learners per class at this school was 80. There were 7 teachers who resided in the schools' residential houses, the rest of the teachers commuted from a radius of about 2 Kilometers. During recess at this school, teachers had in-service trainings in different areas like teaching and learning methods of different subjects and school management. Primary Education Advisors and other education officials monitored teaching and learning at this school at least once in a month.

(ii) Primary School B

The school was located in rural area. It had adequate text books like teachers' guides and learners' books. During time of data collection, four out of nine teachers resided in school houses. The rest commuted from a distance of about 12Kilometers. The school had both experienced and inexperienced teachers.

(iii)Primary School C

This school was also located in urban area. All 28 teachers at this school were qualified and had taught for more than 5 years. There were also only five teachers who resided at the school houses, the rest commutated from a maximum distance of 5 Kilometers due to inadequate houses. Teachers were supervised by education officials at least once a term at every term.

(iv) Primary School D

This was another rural school, many teachers operated from the urban area except the head teacher because the school had one teachers' house. One teacher cycled 15Kilometers to school every day. The school is situated at a Teacher Development Centre. Teacher Development Center is a place where teachers from about 10 to 15 nearby schools meet for continuous professional development activities such as in-service trainings and activities. The school had newly recruited and experienced teachers.

(v) Primary School E

This was another rural school located6 Kilometers away from the main road. Two out of ten teachers at the school commuted from a distance of about 20Kilometers. The school was rarely supervised by education officials (about once a year) may be because it was hard to reach. The school also had both, experienced and inexperienced teachers.

3.4 Data collection

Data were collected using a test, demographic data form and the focus group discussion guide. In the following subsection, I will describe each data collection instrument.

(i) Test

A test comprised of fifteen items was developed. A test was used to assess MKT of teachers. Eleven of the items were adapted from Learning Mathematics for Teaching Project (University of Michigan, 2013), one item was from Mamba (2011) and the

remaining three items were adapted from Measures of Effective Teaching Project (University of Michigan, 2012). Kazima, Jakobsen and Kasoka (2016) validated the use of Learning Mathematics for Teaching Project items in the Malawian context to measure teachers' MKT. Out of the fifteen items; six were testlets (items with operational meanings and sub-items). Since some items were testlets, in total there were 33 items. Each item had one score, therefore the test had a total score of 33. Table 3.2 shows number of items under each algebraic concept and corresponding score.

Table 3.2: Score % for each algebraic concept

Algebraic concept	**Item no.**	**Score**	**% Score**
Equal signs	1a, 1b, 1c,1d,1e &2	6	18.2
Patterns	6a,6b,6c,6d,8 &13a,13b,13c&13d	9	27.3
Variable	4,7a,7b,7c,7d,9,10,11, 12a,12b,12c,12d&15	13	39.3
Linear equations	3a,3b,3c,5&14	5	15.2
Total		**33**	**100**

Table 3.2 shows that the concept of variable had the highest score of 39.3%. This is so because the concept encompasses many topics of algebra in the Malawian Primary school mathematics curriculum. Patterns had second highest score of 27.3% because it is also taught in every class at primary school. Equal signs and linear equations had least scores of 18.2% and 15.2% respectively because equals sign is mostly taught in

lower classes (first to fourth years of primary school) while equations are taught in eighth year of primary school in Malawi.

(ii) Demographic data form

A demographic data form was used to collect demographic data for teachers. The data included participants' academic and professional qualifications. Each participant filled this form which was attached to each test paper (see appendix 1).

(iii) Focus Group Discussion guide

At each school immediately after administering the test, all teachers were asked to put their pencils away and participated in focus group discussions (FGD) which lasted for about 35 to 45 minutes. The FGDs helped to gather more data regarding how teachers got their responses. In other words, FGDs helped triangulate the findings. The FGD guide, directed the discussion between the researcher and the teachers (see appendix 2).

3.5 Pilot study

Clark-Carter (2004) defines pilot study as a trial run of the study conducted on a smaller sample than that to be used in the final version of the study.

Clark-Carter (2004) further argues that the purpose of a pilot study is to check that the basic aspects of the design and procedure work. For example, whether participants understand the instructions they are given and whether the items are valid. In addition, pilot study helps to determine how long the procedure would take with each participant.

In this study, a pilot was conducted to six teachers (three from a rural school and the other three from an urban school). All were mathematics teachers for either standard 5, 6 or 7. All teachers sat for the test at the same time, and soon after the test, they participated in FGD. To ensure reliability, I asked colleagues to check the test items if they were suitable for the teachers. The demographic item form which was attached at the back of each test paper was piloted as well.

The pilot study helped to determine amount of time teachers would need to answer the test and further, the pilot study assisted to polish the FGDs guide to suit the context of the teachers.

3.6 Data collection procedures

The actual data collection was done from second week of third term in April, 2016 to May, 2016. Currently, in Malawi, the primary school calendar has three terms. First term begins in Septembers and ends in December. Second term runs from January to March and third term is from April to July.

At each of the five schools, the teachers were supposed to answer the test items, fill demographic data form, and there after participate in FGD. Prior to data collection, at each school, I phoned the head teacher to inform him/her about my visit. Upon my arrival at the school, I introduced myself, showed the letter from my supervisor to allow me conduct the study and stated the objectives of the study.

Before the participants started answering the test items, I outlined the purpose of the study and demonstrated how to answer the test items. To assure confidentiality, each teacher was given a code which was used on the test paper for identity instead of their real name. At each school, immediately after administering the test, all teachers

participated in focus group discussions. During FGDs, teachers explained item by item how they arrived at the response, then as a group they agreed on the correct responses. To avoid cheating (rubbing the original option and circling the correct response after discussing the item), I asked and made sure that the teachers kept away their pencils. During the FGDs, I audio recorded all the discussions. In the evenings of each day, I transcribed the recordings. On average the FGDs lasted for 40 minutes.

3.7 Data analysis

Quantitative data in the form of test scores were captured for each respondent, anonymised and analysed using Microsoft Excel. Data were backed up in compact disc and e-mail for proper management. Measures of central tendency particularly mean were calculated. Frequencies were used to generate patterns and generalisations were made and interpreted.

Qualitative data inform of transcripts were produced by transcribing the audio recorded FGDs. The data were analysed by studying and identifying common themes among the FGDs.

3.8 Ethical procedures

Before any data collection was carried out in any school, permission was sought first from the District Education Manager. After this permission was granted, further permission was requested from the head teachers. Each participant was assured of the highest confidentiality of the responses obtained from the study. It will be observed that even in the reporting of the findings, no names are mentioned in order to protect

the identity of the respondents and the schools. Further to that, each participant was allowed to withdraw anytime s/he felt so.

3.9 Chapter summary

The chapter has discussed how data were collected and analysed. Mixed method was used; a test was administered to teachers followed by focus group discussions. The chapter has also discussed the sampling and data collection procedures, and how the pilot study was conducted to ensure reliability of the test items. Finally, data collection procedures and analysis have also been described. In the next chapter I will present findings and discuss results of the study.

CHAPTER 4

RESULTS AND DISCUSSION

This chapter presents and discusses results of the study according to the research questions. The supporting research questions are:

1. What characterise Malawian primary school teachers MKT of early algebraic concepts?
2. Are there differences in MKT of early algebraic concepts between teachers of urban and rural schools?
3. Are there differences in MKT of early algebraic concepts between groups of experienced and inexperienced teachers?

4.1 Research question 1: What characterise Malawian primary school teachers' MKT of early algebraic concepts?

As stated earlier, a test comprised of four algebraic concepts was administered to teachers in each of the five sampled schools. The concepts are equals sign, patterns, variable and linear equations. Attached to the test paper was a demographic data form which each teacher filled. Immediately after administering the test, at each school, the teachers participated in FGDs. In the sub-sections below, I will present and discuss results for each algebraic concept.

4.1.1 Teachers' MKT about the concept of equals sign

Items1 and 2 of the test, assessed teachers' MKT about the concept of equals sign. Item 1 was a scenario of a teacher who found a supplementary worksheet in one of the books at their school library. The worksheet asked learners to fill in one or more blanks in a mathematical statement with numbers that would make the statement true. The item was a testlet and had five items numbered 1a), 1b), 1c), 1d), and 1e). Teachers answered the items by ticking against each statement to indicate whether it was mathematically problematic, not mathematically problematic or indicate if they were not sure of the correct response.

Item 2 was a setting about a teacher who was concerned with his learners who viewed the equal sign as a signal to carry out an operation rather than as a symbol indicating equality. The item had missing number problems and demanded that teachers (participants) choose the best example out of the four given examples which they would use to best assess whether their learners understood correct meaning of the equal sign.

Table 4.1 shows number and percentage of teachers who gave correct responses to items 1 and 2. The table shows that in each testlet of item 1, less than 50% of teachers gave correct responses to the items for testlet 1 correct. In addition, the table shows that some teachers gave correct responses to item one and failed other items of the same item. For example, 9 teachers gave correct responses to item 1a) while 14 teachers got item 1e).

Table 4.1 also shows that 51% of teachers gave correct responses to item 2. The demographic data showed that none of the teachers had attended in-service training on the concept of how to teach equals sign.

Table 4.1: Number of teachers who gave correct responses to the items about the concept of equal sign

Concept of equals sign	**No. of teachers who gave correct responses to items 1 and 2 (N=35)**	**% of teachers who gave correct responses to items 1 and 2**
Item 1a	9	26
Item 1b	10	29
Item 1c	11	31
Item 1d	11	31
Item 1e	14	40
Item 2	18	51
Mean number of teachers	12	35

During FGDs, some teachers' responses were mathematically incorrect. For example, at school C, the teachers said:

> *We don't have any difficulties with this problem. A learner can solve them. That item is a pattern (referring to item 1b); the next term would be 29+5*

In addition, at School B, the teachers said:

Equals sign may mean next step and it is a result of, so what is wrong with these problems? We have used that several times with our learners in class

Other teachers said all problems were mathematically valid because learners would solve them, however, some learners would have problems with some items because they were difficult to solve. At School D the teachers said:

There are no issues with these items only that some problems are difficult to solve; some learners will fail to solve them like number 1c

Responses to item 2 also indicated that some teachers gave correct responses to the item by chance. This was evidenced during FGDs; some teachers failed to give mathematically correct reasons about how they arrived at the correct response. In all FGDs they were some teachers who admitted that they guessed the answer since the test was multiple choice. For example, at school B, the teachers said:

I chose response A to item 2 because it has two gaps. Therefore, since there are two equal gaps, so we can have 9+9. And these are the only numbers because we are talking of equality

I further asked the teachers if they learnt about equals sign during pre-service training. They indicated that they did learn on the topic of 'teaching addition of whole numbers'. Teachers of School A said:

We teach learners in lower classes that equals sign mean 'zikhala' (meaning is a result of) and this is what we learnt during pre-service training.

I further asked teachers the kind of reference materials they use to teach the concept of *zikhala*. In all FGDs teachers said, they did not use any reference material other than the teachers' guides and learners' books. Furthermore, School B did not have standard 7 mathematics teachers' guide. In Malawi, a teachers' guide is a text book recommended by Ministry of Education to be used for teaching in primary school for a specific class. A learners' book is a book with activities which learners are supposed to do. Activities in the learners' book correspond to activities in the teachers' guide. Each class and each subject at primary school has a teachers' guide and learners' book.

In all FGDs, at the end of our discussion of items 1 and 2, the teachers indicated that the test items and the discussions were an eye opener. For example, teachers of School, A said:

> *We never thought that equal signs also mean equality. This is true. We just use them anyhow, we never thought much about them*

The findings of the test and FGDs for items 1 and 2 revealed that the teachers did not have adequate knowledge about equals sign as a symbol indicating equality.

Teachers' lack of MKT about the concept of equal signs could be attributed by a number of reasons. Firstly, during pre-service the teachers did not learn that equals sign may also indicate equality. Instead they learnt little about equals sign when learning the concept of basic operation of whole numbers. However, during the stage the teachers learnt equals sign as a signal to carry out an operation. In Malawi, all pre-service teachers are trained on the same curriculum, in view of this; I concluded that

most teachers did not learn about equals sign as a symbol indicating equality during pre-service training.

The second reason that might have contributed to teacher' inadequate MKT of the concept of equals sign might be that equals sign do not appear as a topic or concept to be taught in current primary school mathematics curriculum (Malawi Institute of Education, 2007; Malawi Institute of Education, 2008). In Malawi, all public primary schools use the same mathematics curriculum. Currently, learners are introduced to equations without teaching them that equals sign may also indicate equality (Ministry of Education, 2005).

Thirdly, teachers had inadequate MKT in equals sign as a symbol indicating equality may be because none had attended in-service trainings on uses of equal signs as evidenced by the demographic data. Lastly, teachers did not have adequate reference mathematics books. Their main reference book was the teachers' guide which is prepared in line to the syllabus. In addition, some schools did not have some mathematics teachers' guide. Helga and Palsdottir (2015) argue that teachers' guides play an important role in teachers' professional development.

The findings of the study are consistent with findings by Asquith et al. (2007) who conducted a study on teachers' knowledge of learners' understanding of Algebraic concept of equal sign and variable. The study revealed that teachers rarely identified misconceptions about either variable or the equal sign as an obstacle to solving problems that required application of these concepts. In addition, the findings of this

study agree with the findings of Ball et al. (2008). They argue that one way of attaining SCK is through trainings.

The findings are also supported by Haylock (2010). She argues that most teachers do understand the idea of equals sign as next step. This idea may result into misconceptions of equals sign. She describes uses of equal signs as ‘as a result of ‘and showing equality. She argues in algebra it is important that learners understand equals sign represent equivalence. She says: ‘In algebraic statements it is the idea of equivalence that is strongest in the way the equals sign is used’ (Haylock, 2010, p252). Further, Haylock (2010) encourages teachers to start teaching the idea that the equals sign means ‘is the same as’ at an early stage even when working on results of calculations.

Furthermore, Cockburn and Littler (2008) argue that amongst algebraic misconceptions which learners develop is the use of equals sign to indicate equality. Learners generalise this from arithmetic. Later on when they are introduced to algebra they develop misconceptions. However, the difference is that in this study, it is the teachers who had misconceptions. Possibly one may conclude that the learners’ misconceptions discussed by Cockburn and Litter (2008) might be a reflection of teachers’ misconceptions.

4.1.2 Teachers’ MKT about the concept of patterns

The test had three items on the concept of patterns (items 6, 8 and 13). Item 6 was a scenario where a teacher prepared a task for the learners. The task involved calculating perimeter of 100 hexagons lined in a row. The teacher knew how to find

the answer but she wanted to anticipate what some of her learners would come up with. The item had 4different responses and the teachers (participants) were supposed to indicate by ticking whether each response was correct, not correct, or not sure of the two. The responses were numbered 6a), 6b), 6c) and 6d). Each correct response was worth one mark.

In item 8 was a situation where a teacher wanted to give learners a problem from teachers' guide. Unfortunately, the teachers' guide did not include an answer to the problem and the teacher wanted to make sure she knows the answer before she works on it with her class. The problem had different shapes which formed a pattern; the first shape was a triangle, then a rectangle, thirdly a hexagon, fourthly a pentagon, then a triangle again and so on. The problem required calculating the 83^{rd} shape of the sequence. Teachers (participants) were supposed to circle one correct answer from a set of four responses.

Item 13 was a setting where a teacher asked learners to develop a rule to predict number of cube faces exposed (including the bottom) when they are placed in a train of cubes of any length. The learners produced different answers. Teachers (participants) were expected to indicate by ticking *yes* or *no* for correct or wrong answer respectively against each response, or indicate *not sure*. Item13 had four responses numbered (13a, 13b, 13c and 13d). Each correct response was worthy one mark. Therefore, total score for items 6, 8 and 13 was nine. Table 4.2 shows number and percentage of teachers who got items 6,8 and 13 correct.

Table 4.2: Number of teachers who gave correct responses to the items about the concept of patterns

Concept of patterns	**No. of teachers who gave correct response to the items (N=35)**	**% of number of teachers who gave correct responses to the items**
Item 6a	10	29
Item 6b	8	23
Item 6c	10	29
Item 6d	12	34
Item 8	20	57
Item 13a	9	26
Item 13b	10	29
Item 13c	8	23
Item 13d	14	40
Mean	11	33

Table 4.2 shows that 57% of teachers got item 8 correct. For the remaining items, less than 50% of teachers got the items correct. Demographic data indicated that seven teachers had in-service training on the concept of patterns.

FGD findings for item 6 revealed that some teachers were not familiar with the problem that is calculating perimeter of such shapes. They did not attempt to answer the item. Teachers of school E said:

There are no such problems in the primary school mathematics syllabus therefore learners cannot manage that nor can we.

Teachers of school D commented:

There are patterns in primary school syllabus but not like this one with these shapes, they are there but of different shapes like rectangles, triangles. This shape is Hexagon

Furthermore, the FGDs findings revealed that teachers think there is one formula for calculating measurement of the figure. Teachers of school C commented:

If there are problems we are given one formula for getting one correct answer, I mean learners are supposed to find an answer, not so many ways and answers. How could one problem have several answers this is confusing. In the teachers' guides there are answers and only one answer, not many answers.

When we discussed the problem together almost all teachers in all schools noted that the problem was not difficult to solve and that there are different ways of presenting the solution. For example, teachers of school A:

Oh now we see, there are different ways of finding perimeter of this figure. But what is the recommended method? It looks like what is important is that the method should be mathematically correct. Any method that gives us the correct answer is correct'.

Table 4.2 shows that more teachers gave correct responses item 8 of the items on the concept of patterns. FGDs disclosed that teachers used different strategies to find the

83^{rd} term of the sequence. What was interesting was that several teachers who gave correct response to the problem used the manual strategy. That is, they lined up the shapes up to the 83^{rd} by following the pattern. Eventually they got the right answer but the strategy was too involving and demanded a lot of time. For example, teachers of school E said:

> *We lined up the shapes following the same order to the 83^{rd} shape and found that it was a hexagon*

Teachers of school A, whom most of them failed the question said:

> *We like patterns and enjoy teaching them. However, in mathematics syllabus learners are required to find up to 10 terms only.*

Furthermore, at School C, the teachers said:

> *This problem (meaning item 8) is difficult, some of us even though we teach mathematics but if given a chance we would not. We did not perform well in mathematics at secondary school and if there were options at TTC, we would not study math, but here (meaning at their school) there are few of us teachers and each one has her own class and each of us teachers has to teach all subjects in including mathematics.*

The FGD findings of item 13 were similar to item 8. The findings for items 6 and 8 clearly indicate that teachers did not have adequate knowledge about the concept of patterns. They also lacked problem solving skills. Teachers' inadequate MKT could

be because firstly, they had inadequate CCK. This was evidenced by some teachers who admitted that they did not perform well in mathematics at TTC as well as MSCE Secondly, the FGD findings show that most teachers were familiar with exercises which were in prescribed primary school text books only. They seemed to rely much on the given exercises; any problem outside the text books was irrelevant to the learners and themselves as teachers. This implies that the teachers lacked problem solving skills. Poly (1973) argues that problem solving involves solving unfamiliar problems or finding solutions to problems which have no clear procedure.

Lastly, teachers lacked curriculum content knowledge. In Malawian primary school mathematics curriculum, there are problems of this nature, for example in Malawi Institute of Education (2009), learners are supposed to find perimeter of composite shapes. However, some teachers who were teaching in classes such as standard 6 were not aware that standard 8 learners learn such problems.

The results support findings by Ball et al. (2008) study. They argue that teachers need to have horizon content knowledge. This knowledge helps teachers articulate how the mathematics they teach fits into the mathematics that comes in later years. Teachers need to be aware of what learners will learn in other classes in so doing the learners will be thoroughly prepared for other concepts.

The results are also in line with Mwale-Phiri (2011) who in her study of establishing the mathematical proficiency of standard eight learners in fractions; she found that most teachers considered learners' procedural fluency in fractions as mathematical proficiency. According to Kilpatrick, Swafford and Findell (2001) mathematical

proficiency is a term used to capture all aspects of expertise, competence, knowledge, and facility in mathematics. Kilpatrick et al. (2001) came up with five elements of mathematical proficiency. These are conceptual understanding this is comprehension of mathematical concepts, operations, and relations. Secondly, procedural fluency is the skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Strategic competence is the ability to formulate, represent, and solve mathematical problems. Fourthly, adaptive reasoning is the capacity for logical thought, reflection, explanation, and justification. Lastly, productive disposition is habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

The results may also indicate that teachers do not help learners develop problem solving skills teachers need to expose learners to different and unfamiliar problems in order to develop problem solving skills. However, in order to develop problem solving skills teachers need to have adequate CCK about those concepts as well as pedagogical knowledge which will enable teachers deliver content effectively.

Furthermore, the results may indicate that some teachers teach instrumentally. Skemp (1987) argues that instrumental teaching may result because of several factors including teachers' lack of MKT. I also noted that respondents believed that there was always one way or formula for solving the problem, yet there were several ways. Therefore, were looking for that formula. When the teachers found one, thought the rest of the options were incorrect. Teachers lacked knowledge to appreciate that learners use different strategies of problem solving, what is important is the mathematical validity of the strategies being used.

4.1.3 Teachers' MKT about the concept of variable

In the test, items 4, 7, 9, 10, 11, 12 and 15assessed teachers MKT about the concept of variables. The findings of some items were similar therefore; I will discuss findings of items 4, 7 and 9. Item 4 was a scenario of a teacher who gave learners a problem to find values of $3x$ in an equation. The learners came up with different responses to the question. The teachers (participants) were supposed to choose correct response by circling one of the given four options.

Item 7 was a test-let which assessed teachers' MKT of variable relationship. In the item, learners were asked to write a mathematical statement that represents relationship between number of boys and girls at a certain school given that the number of girls was 50% more than boys. The learners came up with different responses. The participants (teachers) were supposed to choose correct responses of the given four options.

Item 9was about a teacher who asked his learners to write a formula for the perimeter of the rectangle whose width was labelled w and length l. The teacher was surprised by different responses which the learners came up with. The participants were asked to circle one correct response of the given four options.

Item 11 was a setting where a teacher asked learners to create input-output machines that added four to each number entered. The learners produced different machines. The teachers (participants) were asked to circle the correct responses out of the given four which presented correctly the learners work to show $a + 4$ machine.

Table 4.3 shows number of teachers who gave correct responses to the items about the concept of variable. The table shows that all teachers gave correct responses to item 4 except two. Demographic data indicates that the two teachers who faileditem4 had Junior Certificate of Education as their highest academic qualification. Junior certificate of Education is a certificate one obtains after passing exams at second year of secondary school. However, since 2016, the Malawi Ministry of Education stopped administering these exams and consequently issuing certificates.

Table 4.3 shows that less than 50% of the teachers gave correct responses to the items for testlet 7. During FGDs, most teachers showed that they did not understand the relationship between the variables. Teachers of School B said:

> *We always have difficulties with such problems we don't understand them and use them (meaning the variables) interchangeably*

Teachers of school D commented:

> *We have never learnt about problems like this one, even at teacher training college*

Furthermore, the table shows that 74% of the teachers gave correct responses to item 9. FGDs revealed that many teachers were familiar with the problem because there are many items of the nature in primary school mathematics textbooks. Teachers of School B said:

> *This is one of the simplest problems, we have so many of this type in standards five, six and seven*

One main reason that might have contributed to many teachers getting items 4 and 9 correct could be that many teachers were familiar with the items which appear in most primary school textbooks.

Table 4.3: Number of teachers who gave correct responses to the items about the concept of variable

Concept of variable	**No. of teachers who gave correct responses to the items(N=35)**	**% of teachers who gave correct responses to the items**
Item 4	33	94
Item 7a	9	26
Item 7b	13	37
Item 7c	15	43
Item 7d	14	40
Item 9	26	74
Item 10	10	29
Item 11	16	46
Item 12a	5	14
Item 12b	15	43
Item 12c	9	26
Item 12d	17	49
Item 15	16	46
Mean	15	44

On the other hand, some teachers failed to give correct response to item 7 because firstly, as explained earlier, many teachers had poor mathematical background. Secondly, teachers did not learn about variable relationship during pre-service training.

The findings are consistent with Haylock (2010) who argues that learners have misconceptions about variable relationship they tend to use the variables interchangeably. Possibly, learners, misconceptions are a reflection of teachers' misconceptions.

The findings might also be an extension of Mamba (2011) who did a study on misconceptions Malawian secondary school learners have about equations. The study found that learners had misconceptions of equations including that of variable relationship. It might be that the learners developed these misconceptions from primary school.

The findings that teachers did not have adequate CCK, agree with findings by Tanisli and Kose (2013). They did a study to evaluate pre-service primary mathematics teachers' ability to discuss and investigate learners' thinking process about the concepts of variable and equality. They found that pre-service teachers did not have adequate subject matter knowledge and had misconceptions. Since pedagogical knowledge depends on teachers' subject matter, the findings imply the pre-service teachers had inadequate pedagogical knowledge as well.

Further the findings agree with Malawi Institute of Education (2014) who revealed that some teachers who taught mathematics in upper primary school (standards 5, 6 and 7) did not like the subject for various reasons such as negative attitude and inadequate mastery of content. The study further revealed that such teachers lamented that if given a chance to choose subjects to teach, would not teach mathematics.

4.1.4 Teachers' MKT about the concept of linear equations

Items 3, 5 and 14 assessed teachers' knowledge about the concept of linear equations. Item 3 was scenery where a teacher gave his learners a problem during introductory lesson on solving equations. The problem required learners to decide a number that could be put in box to make the given sentence true. Learners got the correct answer but used different strategies. Three strategies were given. The participants were asked to indicate by ticking if each strategy provided evidence of mathematical valid reasoning or not.

Item 5 was a scenario that involved a word problem which a teacher and her learner Chisomo made for the class. The problem was made on Chisomo's birthday who was exactly twice as old as her brother then. The class was asked to calculate when Chisomo would be twice as the brother again and they came up with different responses. The participants were asked to choose the correct response by ticking one of the given four options.

Item 14 was a situation where learners were studying equations. Then the teacher asked the learners to write equations with x and y, such that if x =7, then y =4. The learners came up with a variety of answers. The participants were asked to choose one

response the teacher would accept as correct from the given four responses. Table 4.4 summarises number of teachers who gave correct responses to the items.

Table 4.4: Number of teachers who gave correct responses to the items about the concept of linear equations

Concept of linear equations	**No. of teachers who gave correct responses to the items (N=35)**	**% of teachers who gave correct responses to the items**
Item 3a	24	69
Item 3b	5	14
Item 3c	7	20
Item 5	4	11
Item 14	29	83
Mean	14	39

Table 4.4 shows that 69% of the teachers got item 3a) correct. 14% of teachers got part 3b) and 20% got part 3c) correct.

In all FGDs teachers explained clearly how they got the answer for item 3a). Part a) explained that the strategy used was that the learner used take away method. Learners were supposed to find out what to take away from 10 to get 5. However, though 14% and 20% of teachers gave correct response foritem3b) and 3c), they admitted it was through guess work.

Further, teachers said that many times they do not have the time to check the strategies learners use to get the answer. Teachers also expected learners to use algorithm they are taught in class particularly when working example. Teachers of School E said:

> *Honestly speaking we rarely check how a learner arrives at an answer, we are more interested in the answer whether it is right or wrong, and in most cases we refer to the strategy we discussed during examples. It would take us a lot of time to check each strategy because the classes are big.*

Teachers of School C said:

> *On average we have 120 learners, one period which lasts for 35 minutes is not enough to check strategies learners use to answer questions*

Teachers of School D said:

> *When teaching mathematics, we start with the example then exercise. That is how it is even in the teachers' guides. When working out exercises learners are supposed to follow what we discussed when working out the example. Where can learners get the different strategies?*

On the other hand, teachers of School E said:

> *In real classroom situation there cannot be learners who can explain how they got the answer like the way the tests did. That's for very*

intelligent learners who cannot even be found in their class but may be in private schools for elite people.

Several reasons could be attributed to the poor performance of teachers. Firstly, the teachers had large classes, up to as many as 120 learners in one class. During the time of data collection, in all schools, most classes had more than one hundred learners. Secondly, teachers believe that a learner who is procedurally fluent is mathematically proficient. The findings are coherent with findings by Mwale-Phiri (2011) whose study aimed at establishing the mathematical proficiency of standard 8 learners in fractions in Malawi. The study revealed that in Malawi teachers regarded procedural fluency in fractions as mathematical proficiency.

Figure 4.5 shows that only 4 teachers gotitem5 correct. Item 5 had the least number of teachers who got it correct. Demographic data showed that teachers who got the items correct had other qualifications like banking and some were pursuing studies at Open Secondary Schools.

During FGDs teachers said the test item was difficult. For example, teachers of School D said:

Equation word problems are difficult, I did not attempt to answer the item, if I was given a chance during pre-service training to study subjects of my choice, I would not study math

Most teachers did not understand what they were supposed to find, they failed to devise problem solving strategy. Polya (1945) argues that problem solving process involvesunderstanding the problem, devising a plan, carrying out the plan and looking back. When teaching it is important that learners understand the problem; the

language should be clear and the context familiar to them. Some teachers did not know where to start from to solve the problem.

Many teachers failed this item may be because they did not have adequate CCK of equations and some had negative attitude towards mathematics. They perceived mathematics as a difficult subject.

The findings agree with Malawi Institute of Education (2014) who found that in Malawi, most primary school teachers lack subject matter knowledge of mathematics and consequently lack pedagogical content knowledge.

In concluding research item 1, in the study, I found that generally teachers did not have adequate knowledge of early algebraic concepts of equal signs, patterns, variable and linear equations. In addition, the teachers lacked problem solving skills. This was evident in the way teachers answered the test items and the FGDs. The demographic data showed that most teachers had not attended in-service trainings in early algebraic concepts. The findings may imply that teachers do not teach effectively and consequently affects learners' performance in algebraic concepts.

4.2 Research question 2: Are there differences in MKT of early algebraic concepts between teachers of urban and rural schools?

In this section, I will present and discuss differences and similarities in teachers' MKT of early algebraic concepts between teachers of urban and rural schools.

4.2.1 Teachers' MKT about the concept of equals sign by school location

Table 4.5shows number of teachers according to school location who gave correct responses to items 1 and 2 of the test. As stated earlier the items assessed teachers MKT about the concept of equals sign.

Table 4.5: Number of teachers who gave correct responses to the items about the concept of equals sign by school location

Equal signs	**School location**			
	Urban N=17		**Rural N=18**	
	Number	**%**	**Number**	**%**
Item 1a	5	29	4	22
Item 1b	6	35	4	22
Item 1c	7	41	4	22
Item 1d	6	35	5	28
Item 1e	7	41	7	39
Item 2	8	47	10	56
Mean		38		31

Table 4.5 shows that the mean percentage of number of urban and rural school teachers who got the items correct was 38% and 31% respectively. Demographic data for teachers from both school locations showed that all the teachers had not been trained on equals sign as a symbol indicating equality. During FGDs teachers of school A commented:

I wish you taught us about these equals sign long time ago, preferably at college. But look now, we have misinformed many children. Anyway it's not too late

The teachers' justifications to their responses were almost the same. Almost all teachers admitted they had never thought seriously about equals sign and that the FGDs were an eye opener. I failed to figure out any difference in teachers' MKT of the concept about equals sign between teachers of urban and rural schools.

The findings disagree with findings by Kunje, Meke and Ogawa (2009). They conducted a study to find out factors which affected learners' achievement in English, Chichewa and mathematics. The study revealed that urban learners performed better in Mathematics and English than rural learners. However, they found that other factors affected learners' performance other than the teachers' MKT. These factors are such as parental support and availability of learning resources such as text books.

4.3.2 Teachers' MKT about the concept of patterns by school location

Table 4.6 shows number and percentage of teachers who gave correct responses to the items on the concept of patterns. These were items 6,8 and 12.

Table 4.6 shows that on average, more rural school teachers (35%) got correct items about the concept of patterns compared to urban school teachers (29%).

Table 4.6 further shows that there was big difference in number of teachers who gave correct responses to item 8. Therefore, I will present and discuss more findings of item 8.

Table 4.6: Number of teachers who gave correct responses to the items on the concept of patterns by school location

Patterns	**School Location**			
	Urban N=17		**Rural N=18**	
	Number	**%**	**Number**	**%**
Item 6a	4	24	6	33
Item 6b	4	24	4	22
Item 6c	3	18	7	39
Item 6d	7	41	5	28
Item 8	8	47	12	67
Item 13a	3	18	6	33
Item 13b	4	24	6	33
Item 13c	4	24	4	22
Item 13d	7	41	7	39
Mean		29		35

As stated earlier, item 8 was a situation where a teacher wanted to give learners a problem from teachers' guide. The teachers' guide did not include an answer to the problem and the teacher wanted to make sure she knows the answer before she worked on it with her class. The problem had different shapes which formed a pattern; the first shape was a triangle, then a rectangle, thirdly a hexagon, fourthly a pentagon, then a triangle again and so on. The problem required calculating the 83rd shape of the sequence. Teachers (participants) were supposed to circle one correct answer from a set of four responses.

Table 4.6 shows that 67% and 47% of rural and urban teachers respectively gave correct responses to item 8. In addition, seven teachers in the rural schools had in-service training in patterns. During FGDs, the teachers who had in-service training in patterns said the training was about developing different patterns and completing given patterns. The rural school D was provided with different shapes which teachers could use as teaching and learning resources when teaching completing patterns. Teachers of school D said:

> *We like this problem (referring to item 8), these shapes are like the ones we have. A certain organisation donated us the shapes. They were meant to be used in lower classes (standards 1 to 4) but we use them in other classes too. Learners like using them too.*

Therefore, a reason why more teachers in rural schools gave correct responses to the items on patterns may be because they had teaching and learning resources which facilitated understanding of the concept of patterns. In addition, some rural school teachers had in-service training in patterns.

4.2.3 Teachers' MKT about the concept of variables by school location

Table 4.7 shows number and percentage of teachers according to school location that gave correct responses to the items on the concept of variable.

Table 4.7: Number of teachers who gave correct responses to the items on the concept of variable by school location

Variables	School Location			
	Urban N=17		Rural N=18	
	Number	%	Number	%
Item 4	15	88	18	100
Item 7a	5	29	4	22
Item 7b	9	53	4	22
Item 7c	6	35	9	50
Item 7d	8	47	6	33
Item 9	10	59	16	89
Item 10	5	29	5	28
Item 11	7	41	9	50
Item 12a	1	6	4	22
Item 12b	9	53	6	33
Item 12c	2	12	7	39
Item 12d	9	53	8	44
Item 15	11	65	5	28
Mean		44		43

Table 4.7 shows the difference in number of urban and rural school teachers who gave correct responses to the items about the concept of variables. I will discuss findings of items 4, 9 and 15.

Table 4.7 indicates that all the teachers gotitem4 except two teachers from urban schools. Demographic data indicated that the two teachers who failed the item had Junior Secondary School Education certificate as their highest academic qualification, while the rest of the teachers had Senior Secondary Education Certificate as their highest academic qualification.

Furthermore, table 4.7 shows that most rural teachers gave correct responses to item 9, while more urban teachers gave correct responses to item 15 than rural teachers. During FGD, I discovered that some urban school teachers were going for evening classes at Open Secondary School and that during data collection period they had just learnt about algebraic expressions and equations. Teachers of School A said:

We learnt something like this a day before. It was something like this

It seemed teachers of urban schools especially those who were going for evening lessons at Open Secondary schools had better CCK compared to others. It is common in Malawi for teachers to go to evening classes at Open Secondary schools to upgrade their academic qualifications. As stated earlier, this is so because in Malawi, primary school teachers graduate with a certificate in teacher education which they obtain after two years. The minimum entry requirements into teacher training college before 2016 were a mere pass in most subjects of the Malawi School Certificate of Examination.

The findings are consistent with findings by Malawi Institute of Education (2014) who did a study in Malawi and found that teachers with poor subject matter knowledge had poor pedagogical content knowledge as well. Shulman (1986) argues that strong content knowledge forms a basis for pedagogical content knowledge.

4.2.4 Teachers MKT about the concept of linear equations by school location

Table 4.8 shows number and percentage of teachers who gave correct responses to the items on linear equations. There were three items which assessed teachers' MKT of linear equations. These are item 3, 5 and 14.

Table 4.8 Number and percentage of teachers' who gave correct responses to the items about the concept of linear equations

Linear equations	**School Location**			
	Urban N=17		**Rural N=18**	
	Number	**%**	**Number**	**%**
Item 3b	12	71	12	67
Item 3b	3	18	2	11
Item 3c	6	35	1	6
Item 5	1	6	3	17
Item 14	16	94	13	72
Mean		45		34

Table 4.8 shows 35% of urban teachers and 6 % of rural teachers gave correct responses to item 3c). FGDs revealed that some teachers in urban area had good grade for mathematics at Malawi School Certificate of Education. For example, one teacher of school C had a distinction in mathematics at Malawi School Certificate of Education Certificate.

Table 4.8 further shows results for item 14 that 94% of urban teachers and 72% of rural school teachers gave correct responses. During FGDs, I also noted that some teachers in urban areas were attending evening classes. During the period of data collection, the teachers had just learnt about linear equations. Hence, the item was familiar.

The findings agree with the findings of Ntibi and Edoho (2017). They conducted a study to examine influence of school location on students' attitude towards mathematics and basic science. The results showed that there was no significant difference in mean.

In conclusion, under research 2, I have presented findings and discussed differences in MKT between urban and rural school teachers. Generally, there was little difference in MKT of early algebraic concepts between urban and rural teachers.

4.3 Research question 3: What differences are there in MKT of early algebraic concepts between experienced and inexperienced teachers?

In Malawi, teaching experience is described by number of years a teacher has taught. The first two teaching years is for probation. Currently, newly graduated teachers are

supposed to work in rural areas for five years before posted to urban schools (Ministry of Education, 2006). Chart 4.1 shows percentage of the sample in terms of teaching experience.

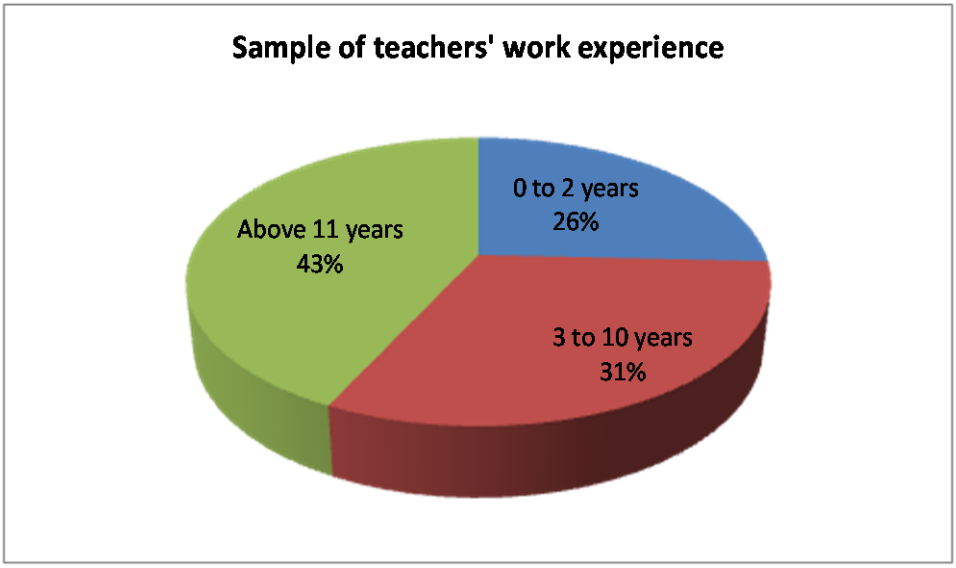

Figure 4.1 Research sample showing teachers' work experience

4.3.1 Teachers' MKT in equal signs by teaching experience

Table 4.9 shows number of teachers who gave correct responses to items 1 and 2

Table 4.9 shows that slightly more teachers with more than 11 years teaching experience gave correct responses to the items. During FGDs, I noticed that more experienced teachers explained the responses with confidence.

The findings are inconsistent with the findings of Ball et al. (2008), who argue that teachers' MKT grow with teaching experience. Probably in this study, both experienced and experienced teachers did not have adequate content knowledge about

equality because the concept is not in the primary teacher education curriculum. In Malawi, all primary teacher training colleges follow the same curriculum.

Table 4.9: Number of teachers who gave correct responses to the items on the concept of equals sign according to teaching experience

Equals sign	Teaching experience					
	0 to 2 Years (N=9)		3to 10 Years (N=11)		Above11Years(N=15)	
	Number	%	Number	%	Number	%
Item 1a	2	22	4	36	3	20
Item 1b	0	0	3	27	7	47
Item 1c	2	22	1	9	8	53
Item 1d	4	44	4	36	3	20
Item 1e	4	44	2	18	8	53
Item 2	6	67	6	55	6	40
Mean		33		30		39

4.3.2 Teachers' MKT about the concept of patterns by teaching experience

Table 4.10 shows that 34% of experienced teachers gave correct responses to the items which assessed teachers about the concept of patterns. During FGDs, more experienced teachers explained that in the past years, there have been items on patterns during end of primary school national examinations. In Malawi, primary school education ends with national examinations. Learners who perform well in the exams are selected to various secondary schools. Teachers of School A said:

Most of us have taught for more than 10 years. During primary school examinations there are always questions on patterns. Therefore, as teachers we need to understand patterns to teach our learners effectively so that they pass exams

In addition, teachers of School C said:

We have been checking primary school national exams; there are always questions on patterns

Table 4.10: Number of teachers who gave correct responses to the items about the concept of patterns according to teaching experience

Patterns	**Teaching experience**					
	0 to 2 Years N=9		**3 to 10 Years N=11**		**Above 11 Years N=15**	
	Number	**%**	**Number**	**%**	**Number**	**%**
Item 6a	3	33	2	18	5	33
Item 6b	4	44	2	18	2	13
Item 6c	4	44	3	27	3	20
Item 6d	2	22	2	18	8	53
Item 8	4	44	6	55	10	67
Item 13a	3	33	3	27	3	20
Item 13b	3	33	3	27	4	27
Item 13c	1	11	3	27	4	27
Item 13d	5	56	2	18	7	47
Mean		19		16		34

The findings agree with Ball et al. (2008) who argue that knowledge of content and learners develop as a result of teaching experience. On the other hand, Malawi Institute of Education (2014) study in Malawi, revealed that teachers who were trained under Malawi Integrated In-Service Teacher Education Program, which was implemented between 10 and 20 years ago, had less PCK in mathematics compared to teachers trained under Initial Primary Teacher Education which is the current teacher training mode, and is implemented since 2006.

4.3.3 Teachers' MKT about the concept of variable by teaching experience

Table 4.11 shows number of teachers who gave correct responses to the items on the concept of variable by teaching experience. The table shows that generally there was small difference between number of experienced and inexperienced teachers who gave correct responses to the items on the concept of variable. Further, the findings show that all teachers got item 4 correct, except two teachers whose teaching experience was above 11 years but their highest academic qualification according to demographic data was Junior Certificate of Education.

The findings agree with findings by Rice (2010). She found that the impact of teaching experience is strongest during first few years of teaching. After that marginal returns diminish.

Table 4.11: Number and percentage of teachers who gave correct responses to the items about the concept of variable by teaching experience

Variables	Teaching experience					
	0 to 2 Years N= 9		3 to 10 Years N= 11		Above 11 Years N= 15	
	Number	%	Number	%	Number	%
Item 4	9	100	11	100	13	87
Item 7a	0	0	3	27	6	40
Item 7b	2	22	4	36	7	47
Item 7c	5	56	3	27	7	47
Item 7d	3	33	4	36	7	47
Item 9	6	67	9	82	11	73
Item 10	3	33	4	36	3	20
Item 11	4	44	7	64	5	33
Item 12a	1	11	1	9	3	20
Item 12b	4	44	5	45	6	40
Item 12c	3	33	4	36	2	13
Item 12d	5	56	5	45	7	47
Item 15	2	22	4	36	10	67
Total	9		11		15	
Mean		40		45		45

4.3.4. Teachers' MKT about the concept of linear equations by teaching experience

Table 4.12 shows number of teachers who gave correct responses to the items on the concept of linear equations.

Table 4.12: Number and percentage of teachers who gave correct responses to the items on linear equations by teaching experience

Linear equations	**Teaching experience**					
	0 to 2 Years N= 9		**3 to 10 Years N= 11**		**Above 11 Years N= 15**	
	Number	**%**	**Number**	**%**	**Number**	**%**
Item 3a	7	78	6	55	11	73
Item 3b	1	11	2	18	2	13
Item 3c	0	0	2	18	5	33
Item 5	1	11	3	27	0	0
Item 14	6	67	9	82	14	93
Mean		33		40		43

I will discuss findings for item 14 because the difference in number of teachers who gave correct responses to the item is big compared to other items. Table 4.13 shows that 67%, 82% and 92% of teachers with teaching experience between 0 to 2 years, 3 to 10 years, and 11 years and above respectively gave correct responses to item 14. During FGDs, I noted that most teachers with more teaching experience explained the way they calculated the item correctly and confidently. This implied that the more

experienced teachers had more MKT then inexperienced teachers. The results agree with Ball et al. (2008) who argue that one source of MKT is teaching experience.

In conclusion, results for research question 3 indicate that teaching experience affects teachers' MKT on early algebraic concepts, which is the more experienced teachers are the more the teachers' MKT.

4.4 Chapter summary

This chapter presented and discussed findings of the study. Generally, teachers had inadequate MKT in all algebraic concepts namely equals sign, patterns, variables and linear equations. The findings also revealed that there is very little difference in teachers' MKT between teachers of urban and rural schools

In addition, in this chapter I presented the findings and discussed that there is difference in teachers' MKT between experienced and inexperienced teachers. The study found that more experienced teachers had more MKT about early algebraic concepts than inexperienced teachers. The chapter ends with a brief summary. In the following chapter I will present summary, conclusion and implications of the study.

CHAPTER 5

SUMMARY, CONCLUSION AND IMPLICATIONS

This chapter highlights important points from all chapters. In particular, it summarises findings and make conclusions. It also states some implications of the study.

5.1 Study summary

This study aimed at investigating characteristics of Malawian teachers MKT of early algebraic concepts of equals sign, patterns, variables and linear equations. In addition, the study intended to explore the effects of school location and work experience on teachers' MKT of early algebraic concepts.

The study was guided by Ball et al. (2008) theory of MKT. The theory categorises teacher knowledge into six domains. For this study, I mainly focused on two domains of the teacher knowledge which are common content knowledge and specialised content knowledge. However, sometimes these domains of teacher knowledge interlock that is there is no clear boundary. Therefore, other domains of teacher knowledge (according to Ball et al., 2008) were discussed spontaneously.

The study generated data by administering a test (comprised of 15 multiple choice items) to standards 5 to 8 teachers in rural as well as urban schools. The items were adapted from the Learning mathematics and Teaching Project, Measures of teaching Project and Mamba (2011). Teachers also filled a demographic data form. At each school, all teachers participated in FGDs immediately after answering test items. The

study revealed that teachers did not have adequate MKT of early algebraic concepts. Most teachers lacked knowledge to anticipate learners' thinking, lacked SCK and lacked subject matter knowledge of the early algebraic concepts.

From literature, work experience and working environment are some of the factors which affect MKT. However, in this study I found that there was not much difference in MKT between rural and urban school teachers. In addition, some urban school teachers had improved CCK because were attending evening classes at Open Secondary schools. Further, some rural school teachers were provided with teaching and learning resources which were part of the test items. This made them be familiar with the problem and had positive attitude towards it hence, they managed to get it correct. Therefore, school location did not really matter but other opportunistic. The study also found that work experience affected teachers' MKT of early algebraic concepts which is consistent to most literature. In Malawi, work experience is defined in terms of number of years a teacher has taught.

5.2 Conclusion

The results of this study have established that teachers do not have adequate MKT of early algebraic concepts of equals sign, patterns, variable and linear equations. Characteristics of teachers MKT of early algebraic concepts include inadequate subject matter knowledge, lack of SCK and lack of knowledge to anticipate learners' thinking.

These characteristics of teachers' MKT in early algebraic concepts could be a results of several factors such as firstly, poor mathematical background. Most teachers had

poor mathematics grades in their MSCE certificates. This made some teachers not to like the subject. However, they still taught mathematics because at primary school teachers are supposed to teach all subjects-there is no specialisation at TTC.

Secondly, most teachers had never attended in-service training in mathematics, instead teachers were using knowledge they acquired during pre-service training. In addition, most schools did not have libraries where teachers would study about contemporary issues in mathematics.

Thirdly, teachers were not conversant with some concepts such as equal signs indicating equality and variable. This was so because the teachers did not learn about the concepts during pre-service nor in-service training. Neither do the concepts appear in primary school mathematics curriculum though they are vital in algebra.

Fourthly, most teachers had large classes of more than 70 learners. This was evidenced during data collection. Apart from of lack of knowledge to investigate learners' problem solving strategies, teachers explained that they had inadequate time to investigate strategies learners used when solving problems.

In addition, most teachers showed that they did not reflect on their teaching practices. This was the case because some teachers did not have enough time to prepare and evaluate the lessons due to long distances they travel to schools and huge work load.
It seems the schools did not have adequate teaching and learning resources including text books and other reference materials. This resulted into teachers memorising

problems in the text books and sometimes failing to apply. Teachers should be provided with different learning materials to widen their knowledge.

Fifthly, lack of teaching and learning resources such as books attributed to teachers' inadequate knowledge about the concepts of early algebra. In some schools, teachers did not even have teachers' guides for mathematics.

Lastly, teachers were exam oriented. What mattered most to the teachers were learners to pass exams especially the national exams. This might have led teachers to teaching instrumentally and not relationally.

5.3 Implications of the study

Based on the findings of the study, the following are implications: Firstly, Ministry of Education should be strict on entry qualifications into TTC. Over the past years, there have been variations in entry requirements into TTC; in some years a credit pass in mathematics was a requirement but in some cases like 2018 a pass in mathematics was not a requirement. The findings of this study imply that pre-service teachers should have good mathematics grade at MSCE.

Secondly, the MOEST should introduce specialisation at TTC. This will help to have mathematics teachers who are interested and good at mathematics. Thirdly, Ministry of Education should encourage in-service trainings, there is need for special programmes of in-service teachers to equip teachers with adequate MKT of early algebraic concepts. During all FGDs, teachers acknowledged that the test and the FGDs were an eye opener; there were some concepts such as equality which they

never took seriously. They admitted during the FGDs, that in the future, they would be more conscious on the way they use equals sign.

Fourthly, there is need to supply schools with adequate teaching and learning resources. In addition, there is need to support in-service teachers who are pursuing studies. This study has revealed that those teachers who were pursuing studies at Open Secondary Schools had improved CCK than others.

Lastly, the study findings imply that teacher-learner ratio need to be reduced. This will help maximise teacher-learner interaction in class. For further research, I propose examining the impact of in-service training on teachers' MKT of early algebraic concepts.

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APPENDICES

Appendix 1: Teachers' demographic data form

1. Participant's number: ----------
2. School: ----------
3. Gender: Male □

   Female □
4. Age: 20 to 30 years

   31-40 years

   40-50 years

   50-60 years
5. Work experience: 0-2 years

   3-5 years

   5-10 years

   Above 11 years
6. Number of years teaching Mathematics: 0-2 years

   3-5 years

   5-10 years

   Above 11 years
7. Current teaching class(es): ----------
8. Highest academic qualification: ----------

   -
9. Professional qualifications/ in-service trainings:

   (i) ----------

   (ii) ----------

(iii) ----------

10. Current positions:

(i) ----------

(ii) ----------

(iii) ----------

Any other comments:

THANK YOU SO MUCH!

Appendix 2:Focus Group Discussion guide for teachers

1. How did you get the answer (for each item on the test)?
2. Why did you choose the responses?
3. Did you learn different uses of equals sign during pre-service?
4. What are the different uses of equals sign?
5. Do you think patterns help learners develop algebraic reasoning?
6. When checking/marking learners work, do you check strategies or methods used to get the answer?
7. When learner writes correct answer but uses strategy you are not familiar with, how do you give feedback?
8. Do you think it is important for primary school learners to learn algebra (for example algebraic expressions, inequalities and equations)?
9. Do you all reside on campus?
10. How far is your house from the school?
11. Are you pursuing any part-time studies?
12. Are teaching and learning resources like textbooks available at your school?