AN EXPLORATION OF TEACHERS' BELIEFS ABOUT THE NATURE, LEARNING, AND TEACHING OF MATHEMATICS IN PART OF ZOMBA URBAN AND BLANTYRE URBAN SECONDARY SCHOOLS

MASTERS OF EDUCATION (CURRICULUM & TEACHING STUDIES-MATHEMATICS EDUCATION) THESIS

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B.Ed (Sciences) –University of Malawi

University of Malawi

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June 2019

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Submitted to faculty of Education, in partial fulfillment of Masters of Education Degree in Curriculum and teaching studies (Mathematics Education)

University of Malawi

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June 2019

DECLARATION

I the undersigned hereby declare that this thesis is my own original work which has not been submitted to any other institution for similar purposes. Where other people‘s work has been used acknowledgements have been made.

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Full Legal Name

Signature

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APPROVAL PAGE

The undersigned certify that this thesis represents the student's own work and effort and has been submitted with our approval.

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DEDICATION

This work is dedicated to my late mother, Rosaline Maseko Tsoka Banda, the one woman that would have been so proud to see me succeed. Your one wish was to see us go very high with our education and I am sure you would have been so glad to see me this far. I never understood why you disciplined me that day when I missed classes until today when I know the fruits of being educated. I miss you every day mum.

I also dedicate this paper to our first born son, Nathan Thanthwe Shawa. Your future is bright son! You will acquire the highest level of education at a very young age. In Jesus' name, Amen!! You will beat your parents' record son.

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ABSTRACT

This thesis discusses a study of Malawian teachers' beliefs about the nature of mathematics, its teaching and learning. A questionnaire adapted from Beswick (2005) with 26 Likert Scale items was distributed to 65 secondary mathematics teachers. Cluster analysis was employed and three clusters of teachers with similar views of mathematics emerged from the data. These three clusters of teachers were named (i) flexible problem solvers, (ii) content understanding, and (iii) teaching for performance clusters. The flexible problem solvers portrayed beliefs about problem solving especially in giving learner autonomy. The flexible problem solvers belief in problem solving faced obstacles when the curriculum prescription surfaced while the content understanding cluster was only concerned about coverage of all topics with accommodation of ideas from fast learners and ignoring slow learners' ideas. The teaching for performance cluster's beliefs were skewed to teacher centred approaches. As compared to the results found by Beswick (2005) in a study conducted in Australia, the results found in this study registered a slight difference. The teachers in the Malawian context showed traits of restriction by the curriculum in their responses unlike the Australian teachers who were not necessarily concerned about coverage of topics. These differences suggested that the context of the teachers' task is what determined their responses. Teachers from the study by Beswick (2005) and this study shared their views in most of the items which had no relationship with the curriculum and had a strong diversion of views for those items that had a direct link with curriculum delivery and time.

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LIST OF ACRONYMS

CDSS	Community Day Secondary School
CERT	Centre for Educational Research and Training
CSS	Conventional Secondary School
DEC	Distance Education Centre
FPE	Free Primary Education
ICMI	International Commission on Mathematics Instruction
IMU	International Mathematics Union
INSET	In Service Training
MANEB	Malawi National Examination Board
MCDE	Malawi Centre for Distance Education
MIE	Malawi Institute of Education
MPRSP	Malawi Poverty Reduction Strategic Plan
NCTM	NationalCouncil of Teachers of Mathematics
SMASSE	Strengthening of Mathematics and Science in Secondary Education
SPSS	Statistical Package for Social Sciences
SSCAR	Secondary School Curriculum and Assessment Reform
TIMSS	Trends International Mathematics and Science Study

CHAPTER 1

Introduction

1.0 Chapter overview

This chapter has seven parts; the preamble, the background of the study, problem statement, research objectives, significance of the study, and the overview of the thesis. The preamble lays a foundation for discussing the background of the study in the next section, then statement of the problem shows the gap in the literature to fit this study, then the general and specific objectives of the study are stated in the research objective section followed by the significance of carrying out this study, and finally, the structure of the thesis is given in the last section of this chapter.

1.1 Preamble

From the International Commission on Mathematics Instruction (ICMI, 1986) document, teachers bring to school attitudes and perceptions about their role in the classroom, their students and also expectations in regards to the curriculum. As such, one of the arguments drawn from the document is that school mathematics can benefit from these beliefs if their perceptions can change and if teachers are assisted to develop necessary new skills that portray the use of problem solving.

The notion of mathematical problem solving was first introduced in mathematics education in 1945 by George Polya in his book '*how to solve it*'. Polya (1945) described the process of problem solving as undergoing four stages; understanding the problem, making a plan of how the problem can be solved, carrying out the plan and finally looking back. From Polya (1945), problem solving is a process of indulging with mathematical problem for non-routine tasks to find the solution. This knowledge was later enriched with Lakatos' contribution whose argument implies that mathematics develops continuously and that it is a process where the solvers indulge in processes of proofs and refutations (Lakatos, 1976). According to Polya (1945), a problem is a task that the solver does not have a readily available route of getting to the solution. Most recently, Skott (2015) argued that if Lakatos' (1976) argument holds water, then there is a need for learners to be exposed to learning activities that allow them to challenge knowledge in activities of inquiry.

Over the years, discussions concerning the shift of teaching strategies from teacher-centred to learner-centred dominated policy and curriculum change towards problem solving by different nations (Xenofontos & Andrews, 2012). In response to this, different initiatives have been put in place to promote teaching with learners at the centre of learning. For instance, reform document from the National Council of Teachers of Mathematics (NCTM) (1991) in the U.S advocated for problem solving. This implies that the classroom environment created by the teacher should allow the learner to be responsible for mathematical knowledge construction in an active process of interaction with their peers collaboratively as the teacher helps the learners in a non-leading manner. Generally speaking, this involves making the learners accountable for their own learning and that the teacher is regarded as a torch bearer and not the custodian of knowledge which imparts mathematical knowledge on learners. One issue that arises in the fight for problem solving is that of teachers' beliefs, which significantly affect the teachers' ways of teaching (Cross, 2009, Fives & Buehl, 2012, Skott, 2009a). This is the case because beliefs in their power as a determinant to practice, may act as either affordances or obstacles (Schoenfeld, 2002).

1.2 Background of the Study

1.2.1 Context of the Study

To define the context of the study, it is necessary to understand the education system of Malawi and the nature of education offered in the schools where the participants for this study were solicited.

From the National Education Sector Plan (NESP) for the period between 2008 and 2017, there are three levels of education in Malawi offered by the government of Malawi namely; the primary school level, the secondary and tertiary level of education (Ministry of Education Science and Technology, 2008). The pre-school is offered mainly by private organizations and individuals, with little participation of the government through the ministry of gender and social welfare. The primary level has eight years and each level is called a standard while the secondary level has four years with each year called a form. Secondary schools are in four groups; Conventional Secondary Schools (CSSs) (which were converted in 1998 from government supported secondary school), Community Day Secondary Schools (CDSSs), grant aided Secondary Schools (basically those secondary schools that are run and supported by denominations), and private schools (those that have zero support from the government but their owners which may be individuals or missions). Furthermore, according to Nampota and Meke, (2014, p.7), grant aided secondary schools are those “whose proprietor is a church but is supported by government in some respects especially in provision of teachers and their salaries”. They further explained that CDSSs are co-owned by the government and the surrounding community of the school while CSSs are fully funded by the government (Nampota & Meke, 2014).

After the introduction of the free primary education (FPE) in 1994 by the Malawi government, there was a great and massive increase in number of learners that enrolled at primary schools. This inevitably led to a large number of learners that transited from primary to secondary school. In response, the manpower at the secondary schools at that time could not carry the mass of students. As such, the government in 1998 converted Distance Education Centres (DEC) which were run by Malawi Centre for Distance Education (MCDE) into CDSS to solve the problem of overcrowding in the secondary schools (Chimombo, Dzimadzi, Kunje & Lewin, 2003). The DECs comprised 'staff trained to teach in primary schools who acted as supervisors to oversee the distribution of learning materials (Mdolo, 2009, p.3).' The CDSSs were still too weak to effectively offer their services due to inadequate number of teachers to teach the large number of learners which led to recruitment of professionally unqualified teachers (Kadzamira, Nthara, & Kholowa, 2004). This is still the case in some of the CDSSs which have teachers that are not professionally qualified. Despite Malawi government's efforts to increase the number of schools to accommodate a good number of students per school or classroom, the problem of overcrowding persisted in the secondary schools as such the government decided to establish the double shifts. In double shift schooling, learners are divided into two cohorts where some attend morning shift and the other group attends the afternoon shift after the end of the first shift (Kadzamira & Hiddleston, 1996). Indeed, both the introduction of CDSSs and introduction of double shifts solved the problem of access but registered another challenge of deteriorating quality of education especially due to insufficient supply of teaching and learning resources (Kadzamira & Hiddleston, 1996). Another characteristic of CDSS is inadequate teaching and learning resources as compared to CSSs. According to Nampota and Meke, (2014) and African Development Fund report (2001), CDSSs and CSSs are distinguished by the availability of teaching and learning resources for the latter as compared to the former. This is so because when the CDSSs were DECs, their financial support was coming from parents and community contributions. Therefore, the change implied that CDSSs financial support too changed onto the government's shoulders with a part remaining to the community. This happened at a time when the government had not calculated the cost and put measures to sort this, as such the government was not well prepared to financially manage CDSSs.

To regain the lost quality of education, the government of Malawi has been involved in different initiatives, and one example of this is a project called Strengthening of Mathematics and Science in Secondary Education (SMASSE) (Nampota & Meke, 2014). Nampota and Meke, (2014) reports that the Malawi government, with financial aid from the Japanese government started SMASSE in 2004 to improve quality of mathematics and science education through offering trainings to in-service teachers. The evaluation for SMASSE In-Service Training (INSET) stressed that its secretariat has been actively involved in the Secondary School Curriculum and Assessment Reform (SSCAR) process, as the experiences of SMASSE INSET have been highly valued. This justifies the dominance of the emphasis on problem solving aspects in the newly adopted mathematics curriculum. It is expected that teachers will embrace it for effective teaching and learning.

1.2.2 The state of mathematics education in Malawi

SMASSE's main focus is to train teachers with an aim of changing their ways of teaching so that they no longer portray the traditional way of teaching mathematics and science (Nampota & Meke, 2014). Contrary to these efforts, there are still foot prints of positivistic view of teaching the subjects. From the project report prepared by Nampota and Meke (2014) for the baseline survey, Centre for Educational Training and Research (CERT) at Chancellor College conducted a study with the aim of exploring in-service and pre-service teachers' pedagogical perceptions, attitudes and practices about secondary school mathematics and science after the phase three of SMASSE project. The study took a broad view of the area by focusing both in-service and pre–service teachers' perceptions and focused on both mathematics and science (Nampota & Meke, 2014). Their participants for the study were teachers and teacher trainers in mathematics and sciences. They checked the correlation between the teachers' beliefs that were inferred from what they said (espoused beliefs) and beliefs inferred from their actions or behavior (enacted beliefs) which they found not to be matching (Nampota & Meke, 2014). Although a majority of the teachers in the study by CERT professed beliefs about teaching and learning which are reform-oriented, their practice portrayed something contrary (Nampota & Meke, 2014). In addition to that, results from a case of a Standard 7 classroom in Malawi studied by Refvik (2014) showed that what is described as problem solving in the literature and what was practiced in this classroom is not the universally agreed understanding of the phenomenon (Refvik, 2014). In other words, his study found no problem solving in the teacher's practices but he did not establish the reason for the observation (Refvik, 2014). Assumingly, could it be the teacher's beliefs connected to cultural context of Malawi which influenced her action? In as much as this is a small case to generalize the findings to the whole country, it is still important to keep this in mind and investigate what beliefs of Malawian teachers define their work. Slightly different from Nampota and Meke's (2014) report, this study focuses on mathematics teachers' beliefs only, specifically practicing mathematics teachers. This angle of study gives specific results for mathematics which allows generalizing the findings to mathematics separately from other science subjects. Because of that, the results of this study act as an informer of what type of beliefs define Malawian mathematics teachers' work of teaching. The instrument that was used in this study was an adapted version of an instrument that was used in Australia by Beswick (2005).

Learners' attitudes are strongly linked with the kind of experiences that their teachers expose them to, which in turn may lead to long lasting belief about the subject and even the content of the subject (Philipp, 2007). Whale (2018) argues that mathematics teachers can impart either positive or negative mindset which can motivate learners to work hard for progress or not. The International Mathematics Union (IMU) (2014) stated that the Malawi National Examinations Board (MANEB) results for mathematics show that most children do not perform well in mathematics as compared to other subjects. According to Kadzamira, Moleni, Kholowa, Nkhoma, Chonzi, Zoani and Chigenda (2004), such public examinations have a role and effect on the way teachers in Malawi teach. They noted that Malawian teachers' practices follow the trends of public examination. According to the SMASSE report by Nampota and Meke (2014), a similar argument is made that the poor performance in mathematics and science is attributed to lack of teaching strategies that puts the learner at the centre of the whole teaching and learning process. In favour of the teachers' practices, IMU (2014) gives a probable reason for teacher-centred choice of teachers to be due to large class size, which constrains and makes it challenging for the teacher to address individual students' needs. Either way, Smith (1996) argues that "teachers of mathematics, like all teachers, need to believe that their teaching actions have significant causal impact on their learners' learning" (p. 393). Whale (2018) calls such action as either bringing about a 'fixed' mindset or a 'growth' mindset.

1.3 Statement of Problem

As with the mathematics curriculum of many developing countries, the Malawi secondary mathematics curriculum has moved away from a focus on skills-and-computation towards more emphasis on the understanding and applications of the basic skills of mathematics (Hau, 1992).

The Malawi Poverty Reduction Strategic Plan (MPRSP), as cited by Nampota (2007), calls for improvement of quality and relevance of secondary science education through provision of adequate teaching and learning resources and appropriate review of the curriculum. To achieve this, the Malawi Institute of Education (MIE) stipulated that its expectation is that teachers will conform to what the curriculum requires of them (MIE, 2013). Thus, to reduce poverty in Malawi using education as a vehicle, teachers have to be the main focus since they are a key factor in the teaching and learning process (Nampota, 2007). According to Wesseler (2005) as cited by Nampota (2007), the teacher factor among other things, include teachers' beliefs about teaching and learning.

Additionally, it is important to check if the teachers have changed their perceptions and practice to teaching and learning mathematics due to the changes in the curriculum or if there are other factors which are still in play. The other side of the call, MPRSP acknowledged the need for provision of enough teaching and learning resources as one way of improving quality of education. However, some schools especially CDSSs are under-resourced in Malawi. Apart from MANEB examinations contributing to the old ways of teaching adopted by Malawian teachers, other contextual constraints include large class sizes which hinder teachers to facilitate learning in an effective way (Kadzamira et al., 2004).

1.4 Research Objectives

The main aim of this study is to investigate what beliefs secondary school mathematics teachers in Malawi hold about the nature, teaching and learning of mathematics. The research questions guiding the study are;

- ✓ What beliefs about the nature, teaching and learning of mathematics do Malawian teachers hold?
- ✓ How similar or different are the results found in my study compared to the findings by Beswick (2005) in Australia?

Basically, the comparison will take problem solving as a nature of mathematics and its corresponding beliefs about teaching and learning will be the point of reference. Thus, teachers' views from the two contexts will be tested if their views agree with giving their learners autonomy and acknowledge use of learner centred approaches as effective ways of teaching mathematics. For a meaningful comparison, this study uses the same instrument which was used in the comparison study done in Australia and the same method of quantitative data analysis which has been elaborated in chapter 3.

1.5 Significance of Study

From the vision 2020, which started in 1997, mathematics is one of the subjects which are believed to help in poverty reduction in Malawi (Nampota, 2007). Although policy makers in the field of mathematics are advocating for problem solving as the most effective way of teaching the subject, teacher beliefs still stand with its other side as a stumbling block to implementing related initiatives (Thompson, 1984). Thus, teachers stand in a dilemma where their beliefs clash with what is expected of them in terms of using problem solving activities. As such, it is important to study teachers' beliefs so that when their beliefs are acting as barriers for change, informed interventions can be employed.

In addition to that, there is an under-representation of the notion of secondary school teacher beliefs in mathematics education (Phillipp, 2007). For instance, until the year 2012, Beswick (2012) noticed that most of the studies in the field of mathematics teachers' beliefs about the nature of mathematics focused on elementary (or primary) school level only. This reveals the gap which shows that there is a need for more studies about teachers' beliefs, especially in the secondary mathematics level.

Furthermore, although many research studies (Andrews & Hatch, 2000; Barkatsas-Tasos & Malone, 2005; Beswick, 2007; Gates, 2006) have been conducted to investigate teachers' beliefs about mathematics teaching and learning internationally, relatively little is known about the Malawian secondary school teachers' beliefs about mathematics teaching and learning. Most of these studies which Beswick (2005) was also referring to are done outside Africa. This is gaining strength from the assertion that beliefs are contextual. Therefore, it is important to study beliefs in the Malawian context as a different context. In so doing, the results from this study will contribute to the broad body of knowledge from other contexts and enrich the literature with findings from Africa, specifically Malawi. The need to conduct this study cannot be over emphasized.

1.6 Overview of this Thesis

This paper has five chapters namely; the introduction, the literature review, the methodology, the presentation, analysis and interpretation of data, and finally the summary of findings, conclusions, and implications. In chapter 1, the introduction which is the current section, begins with laying out the foundation of this research by giving a brief background of the study as its preamble, followed by the problem statement which leads to the purpose of the study. After this, the research questions guiding this study have been stated to show the direction of the study and the significance of conducting this study is then discussed. In chapter 2, the literature review, critically reviews literature on teacher beliefs about the nature, teaching and learning of mathematics. It is under the literature review chapter where the theoretical framework holding this study is discussed. Since teachers' beliefs are argued to be consistent with their practice, their professed beliefs are what they are mostly likely to enact. Another point of discussion is that beliefs and knowledge are not intertwined. In chapter 3, the methodology section, the instrument for data collection is described and also a procedure of data collection and sampling the participants are provided. Chapter 3 also describes the cluster analysis method which was used to analyse the data. Under chapter 4, this thesis presents the findings from the data collected in the survey, and also gives an interpretation of the results from the analysis. In chapter 5, which is the final chapter, a summary of findings obtained from the data collected in this study, conclusion and implications of the results is presented.

CHAPTER 2

Literature Review and Theoretical Framework

2.0 Introduction

This chapter will provide a review of debates regarding beliefs about the nature of mathematics, beliefs about mathematics teaching and beliefs about mathematics learning. The literature review has been divided into subsections with themes of; mathematics teaching and learning, definition of beliefs, context as connected to beliefs, and also the difference between beliefs and knowledge. The literature review will be followed by a theoretical framework for studying mathematics teachers' beliefs about the nature, learning and teaching of mathematics which was developed by Beswick (2005).Under the theoretical framework, there is a discussion of the three categories of the beliefs about the nature, teaching and learning of mathematics and other categorisations of teachers' beliefs by other scholars.

2.1 Literature Review

2.1.1 Mathematics Teaching and Learning

For a long time now, there have been a lot of calls for the shift of teaching mathematics so that it embraces teaching strategies that advocates for social construction of knowledge (NCTM, 1989; 1991). Borgersen (1994), through his study in an exploration of the best way to teach geometry at Agder University in Norway, found out that geometry can best be taught if learners are provided with open ended problems so that its effectiveness in teaching the course should manifest. In addition to that, it was noted that problem solving can best be implemented in small groups because small groups create a non-threatening environment for the learners to participate in their own learning. Furthermore, he recommended that it is important for teachers to emphasize on students capabilities, and to focus on motivating learners in the process of problem solving. But how can mathematics be motivating if it is taught deductively? Roughly, the process of discovering mathematical knowledge does not happen in a single day. Rather, according to Lakatos (1976), initial assumptions undergo different tests where counterexamples are sorted to refute a proposition's weak point. Although this is so, the refutation(s) may be cleared by making an exception of the counterexample in the proposition (Lakatos, 1976). This means mathematics is dynamic, and with continued inventions and re-inventions. This view advocates for the elimination of the traditional way of teaching, where learners are provided with definitions, algorithms and rules to memorize (Freudenthal, 1991, as cited by Larsen & Zandieh, 2008). Rather, gives chance to learners even if they encounter errors in their work.

In mathematics education, problem solving is recommended as an effective way of teaching mathematical content (Schoenfeld, 2007). Lakatos (1976) argued that mathematical concepts develop in learners if they are involved in processes of proofs and refutations. Hence an environment where learners are given the chance to explore mathematical ideas by themselves regardless of whether they get stuck or not, struggle a little bit or more is important for their learning to occur. This can be achieved through problem solving methods of teaching. If learners get stuck or face difficulties then the teacher’s role is to encourage them by persistent questioning that will guide their thinking or giving hints without telling them what to do.

2.1.2 The Construct of Belief

The quest for ways to mitigate the traditional way of teaching mathematics through a number of reforms is not new in the field of mathematics education and different researchers have looked at this from different angles (Skott, 2009a). One of the sides of this multifaceted notion is yet another complex discipline of teacher beliefs. According to Skott (2009a), research on teachers’ beliefs was intended to help deal with problems faced during the implementation of “the new and more process-oriented approach to mathematics instruction (p.28)”. In his recent work, he argued that contrary to the purpose of the field of teacher beliefs, it seems to have not fulfilled the promises of its founders and followers (Skott, 2015).Although this is the case, the expectation that it will support curriculum reforms still stands its chances (Skott, 2015) hence the need for continued study of beliefs. However, studying teachers’ beliefs is not as easy as one may think due to the challenges that the field faces (Pajares, 1992). One of the issues faced in research for teachers’ beliefs is the lack of consistency across definitions for ‘belief’ (Pajares, 1992; Philipp, 2007). In spite of this challenge, research in this field is done with review of different definitions by different previous researchers which at the end the current researchers are left with a task of choosing a definition of best fit for their study (Pajares, 1992). Skott (2015) argued that, although there are inconsistencies in defining beliefs by different researchers, there are always intersecting points for these definitions. The definitions capture the aspect of beliefs’ subjectivity and that beliefs are held cognitively by individuals. In addition to that, Skott (2015) continued that beliefs are subject to change with time though in some contexts where the beliefs are stable may imply long term change. Skott (2015) argued that the definitions of belief also affect the way individuals behave. Below is a brief review of some of the definitions by different scholars. From this discussion, there will be the task of relating some of the conceptions in literature and drawing a working definition for this study with its justification. The issue of distinguishing beliefs and knowledge, which also falls under defining beliefs (Skott, 2015), will later be discussed in the literature review. Nevertheless, let it be mentioned that this research will consider beliefs and knowledge as unrelated constructs.

As a point of departure, Phillip (2007) defines beliefs generally, as “lenses through which one looks when interpreting the world” (p.258). This definition in itself captures the focus that beliefs define the kind of behavior of a person or the actions that they are probably going to manifest. Rokeach in his definition suggests that a person can be aware of their beliefs or not and that this is “inferred from what they say in any simple proposition or actions” (Rokeach, 1968, p. 113).

Artzt and Armour-Thomas (1998)’s definition of teachers’ beliefs describes teachers’ beliefs as being personal and as assumptions about the nature of their subject, its teaching and learning. That is, teachers’ beliefs of a certain subject like mathematics, include teachers’ ideas of what they think mathematics is and what they consider as effective teaching strategies. From these definitions of belief, teachers’ statements, whether written or spoken, are what they believe and their agreement or disagreement to certain assertion can be used to determine their degree of their belief to the topic in question. This therefore validates the use of likert scale questionnaire for collecting data regarding beliefs. Briefly, Pajares (1992) defines beliefs as “…an individual’s judgment of truth or falsity of a proposition" (p.316). This definition by Pajares (1992) may be considered as a summary of the other definitions provided by the above stated researchers because an individual's judgment may be verbal or written, may give an interpretation of the world, may predict the holder's actions, and that a judgment can be passed consciously and unconsciously.

From these definitions, I derived a working description of teacher belief which best suits the nature of this study. This study considered beliefs from Pajares' (1992) definition of belief stated above with modifications which have been contributed by parts of the other definitions discussed above. The working description of this study is as below:

Teacher's belief is an individual teacher's conscious or unconscious judgment of truth or falsity of a proposition inferred from what they say, in spoken or written words, through which one can see their perceptions of the nature of their subject that is most probable to manifest in their teaching and learning process.

This working description best suits this current study because it talks of judging what is true or false about a proposition. In this study, teachers responses to the propositions captured in the Likert scale items are regarded as their beliefs. Additionally, it captures the part which beliefs can be pulled out from individual's writing from which one can easily know what to expect from their practice without the actual observation of their teaching.

2.1.3 Teachers' belief versus practice

To help solve the problem of teaching mathematics in a traditional way, the mathematics curriculum for secondary schools in Malawi aims at investing in the human capital to produce independent citizens that can apply problem solving to their everyday lives and challenges in the nation (MIE, 2013). However, one may ask if curriculum change alone is enough to yield desired results to change secondary mathematics teachers' practice in Malawi.

In 1987, the mathematics curriculum for both the primary and secondary school underwent a major review in Malawi. The aim for the review was to instil skills that align with the role of mathematics, thus to develop mathematical skills that can be applied in learners' lives outside the classroom and to solve the problems which learners encounter by replacing the traditional way of teaching to learner-centred teaching strategies (MIE, 1990 as cited by Hau, 1992). In 2003, the mathematics curriculum was seen to be more skewed towards creating thinking students with more attention being given to mathematical processes rather than emphasizing on test scores (MIE, 2003). Further to this, the reviewed mathematics curriculum which is in its fifth year of implementation places more emphasis on problem solving using collaborative teaching methods, communications, mathematical reasoning, and mathematical connections and representations (MIE, 2013). An official mathematics teaching syllabus for secondary schools states that "learners develop such skills as computational, reasoning, critical thinking, and problem solving through the learning and application of mathematics" (MIE, 2013, p. xi).

Despite the Ministry's efforts to de-emphasize the focus on examinations, the orientation by schools and the general public towards examinations over the years does not seem to have decreased. There is a perception by teachers that public examination results reflect their ability as teachers to teach (Mdolo, 2009). It has been noted that teachers' ways of teaching in Malawi are examination oriented (Mbano, 2003, as cited by Mdolo, 2009). This is so because their ability to teach is attributed to good performance of their learners during MANEB examinations, as such this affects the way the curriculum is implemented in the classroom. In addition to that, good schools are often labelled as those which are able to produce good results in the public examinations just as Cheah (2010) observed in Malaysia. In addition to the other flaws registered by the public examinations as a measure for learners' achievement, well performing schools in Malawi's public examinations, usually schools in urban areas, are labelled as good schools (World Bank, 2010). Because of this, teachers are committing 'too much' of teaching and learning time to the mathematics that appear on tests similar to the case in the U.S. (NCTM, 2006). According to Beswick, Callingham and Watson (2012), the implication of this type of teaching is the production of less effective results especially in mathematics.

This study comes at a time when the newly developed or 'problem solving oriented' curriculum is out and ready to be implemented at full scale. Ernest (1989b) argued that curriculum change alone cannot lead to successful dominance of problem solving. Rather, there is a need for understanding teachers systems of beliefs to compliment curriculum change (Ernest, 1989b; Cross, 2009). According to Ernest (1989b), initiatives and curriculum change that promote problem solving alone may be in vain if teachers' practices are not changed. In his argument, Ernest (1989b) suggested that if problem solving is to fully dominate, it is important to focus on teachers' practices so that they embrace the expected. This can only manifest if teachers systems of beliefs about the nature of mathematics, teaching and its learning tally with the reform (Ernest, 1989b; Cross, 2009).

The argument which states that teachers' beliefs are a determinant of classroom activities is not new in the field of mathematics education. It has been discussed, debated and researched for so long and has stood its ground until today. This conception even finds a place in the definition of the phenomenon (refer to the discussed definitions in subsection

2.1.2 above). Therefore, understanding the drivers of teachers' behavior may inform behavior change measures aimed at best teaching strategies. For instance, some researchers have shown that teacher's beliefs are a determinant of their actions in class (Tsangarodou, 2008) even though others argue the converse (Guskey, 1986). In a study of two trainee teachers of physical education, Tsangaridou (2008) found that the teachers' practices were directly related to their professed beliefs about physical education while Guskey (1986) argues that teachers' experience in implementing new teaching strategies is what changes their beliefs and attitudes. Thompson (1992), as cited by Buehl and Beck (2015), holds a slightly different view that the relationship between beliefs and practice is a two-way relationship. That is, on top of beliefs being a determinant of practice, Thompson argued that practice is also a factor of what beliefs one may hold (Buehl & Beck, 2015). Yet others have found that beliefs are disconnected from their practice and that there is no relationship between the two (Lim & Chai, 2008, as cited by Buehl & Beck, 2015). This study adopted the argument which considers beliefs as a factor for practice. This best fits the data collection tool used in this study and has a backing for its validity from Leatham's (2006) sensible systems framework discussed below. Thus, regardless of the so called 'incongruence' that occur between espoused (professed) and enacted beliefs (beliefs in action) and the controversy around this debate, beliefs have an extent to which they affect teacher's decision about instruction (Buehl & Beck, 2015; Leatham, 2006). Hence, there is a high likelihood of teachers practicing what they believe and a high correlation between professed and enacted beliefs.

The sensible systems framework as proposed by Leatham (2006) holds the view that teachers are sensible beings so that what they believe is what they practice. In his writing, he states that one other assumption that this framework is built on is the assumption that "what one believes influences what one does (p.92)". In addition to this, if researchers find inconsistencies between espoused and enacted beliefs then it is due to either their failure to see the belief which correlates with an observed action or the inconsistencies (Francis, Rapacki, & Eker, 2015). In this framework, teachers may not be conscious of what they believe in but their actions may help the researcher infer what belief is influencing an observed action. Leatham's framework only helped in validating the data collected for my study since my study was conducted with the anticipation that whatever response teachers give can be used as a window to see through what possible ways the teachers can teach mathematics even without actually observing their lessons.

In essence, teachers' beliefs are organized in a coherent manner so that when beliefs that may be seen as contradictory come together, the individual holding them finds a way to resolve the two within the system until coherency is reached (Francis et al., 2015). From this system, the assumption is that where teachers may choose options which may seem contradicting, they may not necessarily be opposing but individual teachers mean what they say and may act likewise for stability and without inconsistencies.

2.1.4 Beliefs as a Function of Context

Teacher beliefs are shaped by different factors; some of which are professional development (Van Zoest, Jones & Thornton, 1994, Buehl & Beck, 2015), level of expertise (Buehl & Beck, 2015), personal factors and environment which surrounds their school (Tschannen, Salloum, & Goddard, 2015). Thus, teachers that have gone through a certain level of education may hold a certain belief from the knowledge which they acquire during training (Buehl & Beck, 2015). Buehl and Beck (2015) add that this may also apply for practicing teachers that undergo a certain in-service training. On the other hand, teachers may hold some beliefs due to cultural and traditional influence that apply for their community or any other social setting which makes them to have similar attributes and shared norms. According to Correa, Perry, Sims, Miller, and Fang (2006), as cited by Ghazali and Sinnakaudan (2014), teachers' beliefs about teaching and learning may be shaped largely by culturally shared experiences. As Levin (2015) observes, the contextual nature of beliefs is portrayed in studies which compare beliefs of teachers from different countries around the world. Thus, Malawi and Australia are considered as two different contexts which may register similar or even different beliefs in a certain area. In addition to that, there are some demands which context presses on its teachers and these also define teachers' beliefs about mathematics. For instance, in Malawi, a person is not expected to argue with the elderly or anyone older than them in terms of age. As such, Mdolo (2009) found this belief as one of the constraints which may hinder learners to present a contrary idea from what their teacher says in classroom activities and neither would the teacher expect such. This is hammering on the fact that context is one of the driving forces of both professed and enacted beliefs (Fives & Buehl, 2012).

Levin (2015) referred context to the larger social, political and economic climate as well as the immediate school context. For example, this study focus on two different countries in terms of their social economic status. Thus, Malawi is a developing country while Australia is a developed country. Therefore, studying these two countries will be enriching to the literature concerning the contextual nature of beliefs. In addition to that, context also includes time and place (Green, 1971). Levin (2015) continued that teachers are involved in different interactions with their colleagues, students, and community members in the context where they are found. From these interactions, teachers' beliefs are shaped and also from collective beliefs that emanate from these interactions. Although it is not always that teachers comply with the reform curricula (Levin, 2015), it is expected of them to abide by the goals of a curriculum developed by policy makers. A similar sentiment is made by Tschannen-Moran, Salloum, and Goddard (2015) where they acknowledge the extent of knowledge and how it is to be presented by the teacher as being constrained by the curriculum of a particular country or state. In addition to this, context is very important such that some of the mismatches that occur between espoused and enacted beliefs are as a result of what context they are observed in (Beswick, 2005). That is to say, teachers may believe in something but enact some belief contrary as constrained by a situation or context. For instance, supposedly the political influence on the curricula adopted by different districts of the United States has different impacts of different teachers that have their individualized beliefs but still are tied to do as expected by the collective beliefs (Schoenfeld, 2007).

The Trends International Mathematics and Science Study (TIMSS) (Beaton, Mullis, Martin, Gonzalez, Kelly, & Smith, 1996), conducted in selected countries around the world, revealed that most teachers believe mathematics is essentially a vehicle to model the real world, that ability in mathematics is innate, and that more than one representation should be used in explaining a mathematical concept. Similar findings were observed by Handal (2003) who argued that despite many educational reforms, a large number of teachers still perceive mathematics in traditional rather than in progressive terms. Satterly (1989) argues that teachers have an obligation to cover a considerable amount of the syllabus to prepare those children that are in the grade levels that write national examinations. Satterly continues that this is to be done so that a good number of their children have a high chance of passing the external examinations well (Satterly, 1989).

According to Nathan and Koedinger, (2000), teachers' knowledge and beliefs about mathematics, teaching, and learning determine how they deliver their curricular in class otherwise the converse is likely to happen. This implies that if what the curriculum prescribes and what is practiced by teachers is different, teachers' knowledge or beliefs may be the factor bringing the current state of teachers' behaviour.

The increasing high demand for studies in teacher beliefs is derived from the quest for effective teaching methods that align with the constructivists view (Skott, 2009a). According to Handal (2003), the behaviourist view of teaching mathematics considers learners as passive recipients of knowledge and mathematics as involving drill method. In Handal's argument of behaviourist perspective, procedures are memorized and with practice the knowledge gets established (Handal, 2003). Contrary to the social constructivist view, behaviourists view has an implication that there is one route to get to the solution of a problem. Social constructivists advocate for teaching strategies that incorporate problem solving, group learning and also an environment that makes the learner to be responsible for knowledge acquisition (Handal, 2003). According to Philipp (2007), almost all mathematics educators support the view described in the U.S.NCTM reform documents which assert that there is more to teaching than just telling (NCTM, 1989, 2000). Further to this, effective teaching is argued to depict learning as active knowledge construction supported and directed by social interaction. Thus, effective teachers facilitate learning rather than explicating mathematical concepts and procedures. In recent years, different initiatives have been put in place to overcome those beliefs which, according to ICMI (1986), the perceptions may act as a stumbling block to effective mathematics teaching and learning. This may be the case since teacher beliefs are determinants of teachers' action and decisions in their classrooms. Furthermore, Skott (2015) acknowledges that there is power when a reform is shared by the teachers if its requirements are to be met by the teachers. That is, teachers must embrace the educational reform to meet its objectives; otherwise its devaluation is inevitable.

Teaching strategies that put the learner at the centre of the lesson are highly developed, and while it is technically possible for teachers to employ innovative teaching tasks, in most cases it is not what is seen in practice. For instance, strict use of textbooks as an approach for teaching mathematics is against the reform orientation of teaching and does not promote learner-centred approach of teaching which is encouraged by constructivists (Ernest, 1989a). Similar sentiments are made by Beswick et al. (2012) who contended with the literature which includes use of textbooks as learner-centred but argued that this is associated with less effective traditional teaching. According to Skott (2015), teachers who teach using learner-centred approaches do not present knowledge to learners as a finished product or expect learners to retain the concept exactly and in a verbatim manner. Rather such teachers"…engage the learners in content related processes that may challenge their pre-understanding and link the contents to their everyday experiences in a meaningful way" Skott (2015, p.15). On the contrary, Nampota (2007) noted that most Malawian teachers stick to the less effective traditional ways of teaching subjects like mathematics and sciences in the name of lack of resources. Nespor (1987) suggested that if research about effective teaching strategies is to bring meaningful results, there is need to understand teachers' beliefs about their subjects which are what highly predicts their practice.

Regardless that teachers' thinking may be significant, Thompson (1984) viewed teachers' beliefs as a primary obstacle to change and a major problem to implementation. This shows that there is still a significant need to study beliefs which may give clear guidance on ways to handle negative impact of beliefs to practice. That is, a teacher may *know* for instance, how to teach mathematics using problem solving but practice otherwise as a result of their *belief*.

2.1.5 Beliefs and Knowledge

Different scholars have looked at knowledge and beliefs from different angles of relationship. Some viewed knowledge as being a subset of belief (Plato, 2009, as cited by Skott, 2015) while others looked at the two as being distinct (Phillip, 2007).

According to Phillip (2007), knowledge is consensual or universal where it cannot be disproved while beliefs are prone to rejection. He further argues that knowledge may be subject to debate and may be disproved or others may believe otherwise. Likewise, Pajares (1992) has similar distinction between knowledge and belief, he argues that beliefs are highly personal and that doubting them within oneself is the same as questioning one's sanity. He continues that beliefs exist to an individual analogous to the same way Mount Everest exists and that they are beyond a person's control so that they subjectively become truth to the holder and that they may also be formed without the knowledge of the holder (Pajares, 1992). Song and Looi (2012) conducted a case study of two teachers with markedly different beliefs about student learning, identified through interviews and video-taped professional development sessions. These two teachers implemented the same lesson plan on fractions and division. From their analysis of instructional practices, classroom interactions and student learning, the authors concluded that teachers' innovation-oriented beliefs implement patterns of inquiry-principle-based practices that in turn support meaningful-student inquiry learning (Song & Looi, 2012). Thus, only those teachers that hold on to this belief are the ones that will yield to the same regardless of their knowledge. Although beliefs can change with time, Buehl and Beck (2015) held to the fact that they are more or less stable or consistent within the individual than knowledge. Furthermore, Green (1971) argued that knowledge is externally verifiable whereas beliefs, according to Nespor (1987), are subjective claims an individual accepts as being true.

On the other hand, knowledge is considered as a type of belief depending on its truthfulness (Phillipp, 2007). From the definition of belief which Phillipp (2007) quotes Clement (1999), knowledge is described as a belief that has a hundred percent probability. It is justified and true. That is, against the view that knowledge and belief are disjointed, this view considers the two as related with one distinction on certainty grounds. This side of argument concurs with Plato's definition of knowledge which is referred to as justified and true belief (Phillipp, 2007). This implies that, whatever an individual believes gains the power of being knowledge, if it is justified and established. Otherwise it is just a mere belief.

2.2 Theoretical framework

2.2.1 Beswick's (2005) Categorization of Teachers' Beliefs about the Nature, Teaching and Learning of Mathematics

Ernest (1998), claims that the epistemology of mathematics takes into account issues of the beginning of mathematical knowledge and the development stages which the knowledge undergoes, the basis and process for establishing mathematical knowledge from being subjective to objective, the role of humans in mathematics, and the connectedness or relationship of mathematical knowledge with other human knowledge and experiences.

According to Beswick (2005), there are three categories of teacher beliefs in mathematics education. These categories are;

1. Beliefs about the nature of mathematics – these are the belief about what knowledge is in mathematics
2. Beliefs about teaching mathematics – these are teachers' view of the process of learning mathematics. Their perception about their role in the teaching and learning process
3. Beliefs about learning mathematics – these are learners' expected behaviour by the teacher, the learners' role and what constitutes appropriate and prototypical learning activities

The three categories branches into other three groups. Before these beliefs are discussed in detail it is important to note that teachers' beliefs about the nature of mathematics translate into their philosophies about teaching and learning (Webb & Webb, 2004). According to Ernest (1998), teachers' personal philosophies of mathematics have an impact on the kind of experiences that they expose their students to. A similar observation was made by Cross (2009) who saw that the teaching and learning experiences offered by teachers were a derivative of their view of the nature of mathematical knowledge.

Thus, there is a correlation between beliefs about the nature, teaching and learning mathematics which led to the framework for studying teachers' beliefs by Beswick (2005). A summary of the framework is shown in table 2.1 below. In the table, each row presents corresponding beliefs and each column is a continuum for the respective type of mathematics teachers' beliefs as extracted from Beswick (2005). Therefore, the discussion in this section will address each belief of the nature of mathematics and its corresponding beliefs about teaching and learning.

Table 2.1: Relationships between beliefs; a summary of the framework for studying teacher beliefs about the nature, teaching and learning mathematics (Beswick, 2005, p 40)

Beliefs about the nature of mathematics (Ernest, 1989b)	**Beliefs about mathematics teaching (Van Zoest et al., 1994)**	**Beliefs about mathematics learning (Ernest, 1989b)**
Instrumentalist	Content-focused with an emphasis on performance	Skill mastery, passive reception of knowledge
Platonist Content	Content-focused with emphasis on understanding	An active construction of understanding
Problem-solving	Learner-focused	Autonomous exploration of own interests

Beliefs about the nature of mathematics branches into the instrumentalist view, the Platonist view and problem solving view. The following subsections discuss these categories of beliefs and their related beliefs about teaching and learning.

2.2.1.1 The instrumentalist view and their corresponding beliefs about teaching and learning

Under the instrumentalist view, "mathematics is seen as accumulation of facts, skills and rules to be used in the pursuance of some external end" (Ernest, 1989, p.250). Thus mathematics is regarded as a set with disjoint sets of topics and rules that are utilitarian (Webb & Webb, 2004). Under this view, the teacher takes the role of an instructor whose main focus is to make sure that students acquire skills that will make them pass examinations. The teacher believes that effective teaching includes telling children or spoon feeding (McDiarmid, 1990 as cited by Van Zoest, et al., 1994). The kind of teaching that these teachers use is the one as a process of imposition and not negotiation of ideas (Bishop, 1985). A mathematics lesson with a dominant instrumentalist view includes more emphasis of mathematics as a memorisation of pre-existent formulae which only require substituting figures in place of symbols during mathematical exercises (Van Zoest, et al., 1994). A similar argument is made by Skemp (1978) that for learners to gain mathematical knowledge under this view, they ought to memorise procedures and be proficient in doing computations. Van Zoest, et al. (1994) write that in this view, the teacher re-directs the students into the right solution method when he/she recognizes that the students are going in the wrong direction. This implies that the teacher does not give room to tension and discovery of new solution methods by the students. In a case where the students' solutions are wrong the teacher ignores them (Van Zoest et al., 1994).

In this view, mathematical knowledge is considered as something absolute, its truth is unquestionable, universal, certain and objective. Further to this, it is not concerned about how knowledge came into existence or how it developed to where it is. The role of the teacher under this view is of an explainer (Cross, 2009). Thus, if this point of view can be translated to the classroom setting, teachers will hold tightly to what is written in textbooks (what Ernest (1989b) calls published knowledge) without any modifications to it. In addition to that, such teachers with the instrumentalist view do not engage learners in active knowledge construction but considers learners as passive recipients of knowledge (Cross, 2009). Teachers' evaluation ways focus much on the answer that the learners get other than the process of getting to the solution (Cross, 2009). This means that all that these teachers care is whether the answers that their students get are right and that their procedural fluency is sharp too (Cross, 2009).

2.2.1.2 Platonist View and its corresponding beliefs about teaching and learning

For the Platonist view, mathematics is regarded as being "static but unified body of certain knowledge and that mathematics is discovered and not created" (Ernest 1989b, p 250). Considering Platonist view distinctly, classroom activities that are derived from Platonists include the strict adherence to structure and hierarchy of topics (Cross, 2009). In this regard, effective teaching is supposed to consider connections that exist among pre-existing concepts which must be offered in that manner without any amendments. In addition to organisation of concepts, Platonist view holds the belief that mathematical knowledge which was in ancient times is the same mathematical knowledge which is now and that there is no room for creation, recreation or addition to its contents. In this view, mathematics is believed to be knowledge that exist somewhere and only waits to be discovered other than being created (Webb & Webb, 2004). It does not change, has intersections within itself, is consistent, and has an objective structure (Beswick, 2012). Therefore, the teacher is regarded as an instructor of the knowledge so that the learners capture and gain an understanding of the unified body of knowledge (Ernest, 1989). Unlike teachers in the instrumentalist view who impose ideas on learners, teachers under the Platonist view suggests solutions to learners when stuck (Van Zoest, et al., 1994). The teacher provides the learner with a range of solution methods from which they choose one for themselves (Van Zoest, et al., 1994). Instead of ignoring wrong answers, they accept both right and wrong answers in a non-evaluative way but avoid focusing much on the discussion of the wrong ones.

2.2.1.3 Problem solving view and its corresponding view of teaching and learning mathematics

Under problem solving view, “mathematics is dynamic, continually expanding field of human creation and invention, a cultural product,” and also “mathematics is a process of enquiry and coming to know, not finished product, for its results remain open to revision (Ernest, 1989, p 250)”. Teachers holding this view prioritise learner’s autonomy of the ever changing mathematics knowledge. They use teaching strategies that allow learners to construct knowledge in groups and create an environment that allows the learners to argue and back their reasoning of mathematical concepts. They accommodate needs of both fast and slow learners. For slow learners, they are allowed to struggle in the process of full acquisition of knowledge while fast learners and genius minds are given the chance to create even new ways of getting to solutions of a problem. Under problem solving view, mathematics is considered as an ever changing, work of art of the human mind, continually expanding field of human creation, and also as constructed during social and cultural interactions (Webb & Webb, 2004). Thus, mathematics is a process of enquiry and coming to know, an unfinished product, and that its results remain open to revision located in a social and cultural context. In agreement with the Piagetian (1961) process of equilibrium, Skott (2015) asserts that for learners to engage in problem solving for acquisition of new knowledge, teachers ought to expose and engage learners with experiences that may challenge their prior knowledge. Thus a little tension is necessary during exploration of possible solution paths where learners may get stuck along the way and move on until they get to the solution. Thus, effective teachers should not deny the learners a moment of disequilibrium so that learning can occur, but should facilitate learning (Ernest 1989). In an instance where learners encounter a problem, teachers that fall in this group will not unpack the mathematical reasoning for the problem. Rather they will persistently pose questions to the learners so that they may turn back if they have reached a dead end in their thinking and re-think, thus motivating them in their moments of frustrations (Van Zoest et al., 1994). That is, teachers are considered to be facilitators of learning (Ernest, 1989b). Yackel, Cobb, and Wood (1991) acknowledge communication in class activities where learners are given time to report their conjectures to the class. When their conjectures meet a challenge by the fellow students, the ones that are reporting carry the task of examining their assertions and modifying them where necessary. Otherwise, the conjecture which does not meet a challenge is considered to be true though its truthfulness must be justified by the owner (Cobb et al., 1991). This is where social interaction proves to play an important role in students' learning (Yackel, Wood, & Cobb, 1992). In addition to that, whenever a teacher detects inappropriate solutions, it is the teacher's responsibility to indulge the students in a series of confident and persistent questioning to challenge them so that they discover the errors in their ideas (Van Zoest et al., 1994).

That is, problem solving can be likened to the fallibilists' viewpoint which takes mathematical knowledge as a product of social interaction, capable of being constructed socially and also opens the door to debate and correction (Webb & Webb, 2004). Unlike the absolutists' view that does not create space for history of mathematics, fallibilist acknowledges that mathematical knowledge underwent different stages of development involving correction, theory replacement, and refining of the knowledge leading it into a continuous process of knowledge growth. Furthermore, human beings are regarded as creators of knowledge and mathematics as a product of human invention, dynamic and constructed during interaction (Ernest, 1998). Mathematics is also considered as a body of knowledge that is connected to other bodies of knowledge which is against the absolutists' stand.

2.2.2 Other Categorization of Mathematics Teachers Beliefs

Some scholars have categorized mathematics teachers' beliefs in two categories. For instance, Fives and Buehl (2012) categorized beliefs about teaching mathematics into two areas; teachers' beliefs about specific teaching practices and teachers' beliefs about holistic approach to teaching. The former refers to those beliefs about topics like, cooperative learning, and teaching mathematics, while the latter include approaches like constructivism, and transmission.

Teachers who believe in transmission have characteristics as teachers described in a study done by Chan (2011). These teachers focus on teaching rules of mathematics, algorithms and procedures, are bound by what is prescribed in textbooks. Furthermore, such teachers do not see learners as able to learn independently but learners are seen as having limited information or as blank slates. From a study conducted by Chan (2011), teachers portrayed characteristics which were the opposite of reform-oriented beliefs. Chan (2011) observed that the teachers later switched to the effective way of teaching. Thus, concurrently, the teachers displayed opposing view of mathematics teaching (Chan, 2011). They used lecture method and in some instances, learners were given an opportunity to work in groups so that the learners could share their mathematical thinking.

A study which Webb and Webb, (2004) conducted in Eastern Cape in South Africa, collected data about teachers' beliefs using a Likert scale questionnaire. The questionnaire was administered to 339 mathematics teachers. From this, they found that the majority of the teachers' beliefs were skewed to the reform orientation with a few under the contrary (Webb & Webb, 2004). For those teachers that had a conception of mathematics that it is fallible, their beliefs about the teaching and learning also fell under the corresponding problem solving view where learners actively learn collaboratively for acquisition of knowledge (Webb & Webb, 2004). On the other end, those teachers that held the absolutist view agreed that learners learn passively and on their own with the teacher as the centre of the lesson doing talk and chalk (Webb & Webb, 2004). This effect asserts that their views of the nature of mathematics fit directly into what can be expected in terms of beliefs about teaching and learning. Thus on a continuum for the nature of mathematics with absolutist on the other end and fallibilists on the other, mathematics teaching corresponds to instrumentalists and problem solving oriented teachers respectively (Ernest 1989).

Slightly different results were found in a study by Sinnakaudan and Ghazali (2014) in Malaysia. With an aim of exploring Malaysian primary school mathematics teachers' beliefs about teaching their subject, they found out that most teachers held both beliefs about formal and informal beliefs, while a smaller percentage of teachers believed in problem solving. In the same study, no teacher viewed mathematics as absolute or objective only.

Collier (1972) categorised mathematics teachers' beliefs into two: formal and informal beliefs. Formal beliefs are those in which mathematics is made up of rules and procedures and is concerned with computations to satisfy them, while informal beliefs are those where the holder perceives the subject as dynamic, involved with exploring and being creative to come up with new ideas in the field (Collier, 1972). Thus, concepts are taught instrumentally and the teacher is the centre of the lesson in formal beliefs. Under informal beliefs, mathematics involves problem solving processes, proof and reasoning, communication, and representation (NCTM, 2000). As such, for one to acquire mathematical knowledge, they have to be involved in active process where they are involved in formulation of problem themselves and are the center of the whole problem solving process. Borgersen (1994) in his study found out that open ended problems are the best way to teach mathematics, and mathematics is best taught if learners explore problems in groups where the teacher is just a facilitator and not necessarily the custodian of knowledge. According to Earnest (1989a), due to the nature of mathematics, children must be engaged in problems which will require critical thinking and reasoning in their context, promote invention and discovery of mathematical knowledge, and involving testing and justification of their arguments. Although different researchers use different terms to describe the categories of teachers' beliefs about the nature of mathematics, teaching, and learning shown in this section, the concept in their explanation is the same.

2.2.3 Summary of the categories of teachers beliefs

The instrumentalist view of the nature of mathematics lacks the idea of the connectedness of methods. As such memorization of formula without 'knowing why' may work for the learners to determine whether learning has taken place. In the Platonist view, the missing connection between concepts is now brought into the picture but knowledge is regarded as objective and stable. This knowledge is taken to be pre-existed and only waits to be discovered. Contrary to Platonism, problem solving view opens the room for knowledge to be created or re-created, revised and contributions by other people are found to be relevant. In addition to that, problem solving accommodates the notion of knowledge as a socially and culturally constructed phenomenon which happens during discussions, debate, and investigations that is done in groups.

2.3 Chapter Summary

This chapter was in two fold; the literature review and the theoretical framework. Under the literature review is a review of related literature to this study while under the theoretical framework is a discussion of the theoretical framework that informs this study.

CHAPTER 3

Research Methodology

3.0 Chapter Overview

This section gives a brief overview of this chapter. This section will be followed by the research design for this study which will reveal that this study is a quantitative type of study. A research design describes procedures used in conducting the study (McMillan & Schumacher, 2006). Furthermore, a research design provides insight on how the collected data should be analysed (McMillan & Schumacher, 2006). As such, this discussion will also present the sample and sampling method used in this study, instrument, data collection process, data analysis, issues of reliability and validity of the study. The final section of this chapter will present a summary of the whole chapter of research methodology.

3.1 Research Design

The aim of this study is to explore what practicing secondary mathematics teachers believe in regards to the nature of mathematics, its teaching and learning in the Malawian context. To get to the desired, the study was designed to be a quantitative study. According to Creswell (2009, p.4), “a quantitative research is a means for testing objective theories by examining the relationship among variables which in turn, can be measured, typically on instruments, so that numbered data can be analysed using statistical procedures”. Specifically, this study uses a questionnaire instrument with 26 Likert Scale items and a framework adapted from Beswick (2005). According to Phillipp (2007), a Likert scale questionnaire for studying teachers is used to capture data from a bigger sample. This turned to be a survey design since according to Creswell (2009, p. 145), “a survey design provides a quantitative or numeric description of trends, attitudes or opinions of a population by studying a sample of that population”. In brief, survey design using a Likert scale is the most preferred for this study because it helps collect data from a bigger sample and the items captured different aspects of beliefs about the nature, teaching and learning of mathematics. The nature of this survey is cross sectional since the research is part of assessment in partial fulfillment of a master’s degree hence has a defined time span. According to Creswell (2009), a cross sectional study is the one where data is collected at a certain point in time while a longitudinal study takes a period of time of phases for data collection.

As such, the next section of this chapter provides a discussion of the sample and sampling technique followed in conducting this research that helped to satisfy the research objectives of the study. As explained in section 1.4, the research objectives for this study were to explore the mathematics teachers’ beliefs about the nature, teaching and learning of mathematics in Malawi as compared to teachers’ beliefs for the same in Australia.

3.2 Sample and Sampling Technique

Sixty Five in-service teachers participated in this study. The majority of the teachers were males who were forty five in number representing sixty nine percent. Forty seven teachers, which represents seventy three percent of the sixty five teachers, had a bachelor’s degree,eighteen teachers (27%) had a diploma with none holding a master’s degree. To solicit the participants, the researcher went with a letter of introduction from The University of Malawi, Chancellor College, to all the participating secondary schools. Upon approval, arrangements were done with either headteachers, Deputy headteachers or in other cases heads of science department for respective schools to help with giving the questionnaires to their mathematics teachers that were interested to participate in the study. That is, consent of the teachers in the identified schools was sort, to participate in this research. Questionnaires were not administered to the teachers in one place and not at the same time because Junior Certificate examinations were close and schools were busy preparing for the examinations. Therefore, teachers asked if they could take the questionnaires with them to complete and return later. Therefore, in all the schools, copies of the questionnaire were left with either the head of sciences department or the head teacher for the school so that their practicing mathematics teachers can complete in their free time then submit back to the heads. The questionnaires were then collected on an agreed day. This activity took about three months from May 2016. On plan, data collection was supposed to take a month but the activity coincided with preparations for Junior Certificate Examinations. This led to failure of most teachers to complete the questionnaire in good time.

A purposive sampling technique was employed where the questionnaire was administered to practicing mathematics teachers for 13 selected secondary schools in Zomba urban and 5 secondary schools in Blantyre urban. The sample was purposive to have as many participants as possible in the interest of time and costs. All teachers in the sampled schools were included because of the limited number of mathematics teachers in the schools. Random sampling would have left the researcher with very few participants, and this would have been a problem because the quantitative research design of the study required as many participants as possible. In table 3.1 below, is the list of schools and their total number of mathematics teachers that participated in this study.

Table 3.1 Secondary Schools

District	School	Type of Secondary School	Total number of teachers
Blantyre urban	S1	Grant aided	3
	S2	Conventional	2
	S3	Grant aided	4
	S4	CDSS	3
	S5	Conventional	8
Zomba urban	S6	Conventional	6
	S7	Conventional	5
	S8	Conventional	4
	S9	CDSS	3
	S10	Grant aided	3
	S11	CDSS	3
	S12	Conventional	4
	S13	Private	1

	S14	CDSS	4
	S15	Conventional	8
	S16	Grant Aided	2
	S17	Private	2

There is no classification of the teachers in terms of what grade level they taught because no matter what level they may be currently teaching, they were all employed under the same conditions of employment by the Malawi Ministry of Education Science and Technology (MoEST). As such they all stand the same chances of teaching any level and are not any different because of this. In addition to this, all the school types follow the same curriculum.

3.3 Instrument

A survey questionnaire for measuring teachers' beliefs about mathematics was used in this study. The questionnaire consisted of 26 Likert scale items, adapted to fit the Malawian context from study done by Beswick (2005) in Australia. The options for responses were rated from strongly agree which corresponds to 5, agree corresponding to 4, neither agree nor disagree corresponding to 3, disagree corresponding to 2 and strongly disagree to 1 (refer to appendix B).

Initially, Beswick (2005) took 35 items regarding mathematics teaching from Van Zoerst et al. (1994) and 5 items about the nature of mathematics from Howard, Perry, and Lindsay (1997) which resulted in a 40-item survey. Upon piloting this with 35 teachers, Beswick

(2005) used factor analysis to get similar number of factors (two factors) as found in Howard et al. (1997)'s study. On the other hand, the factors found by Beswick (2005) were slightly different from Van Zoerst et al. (1994)'s number of factors, that is two versus one factor respectively. The factors identified by Howard et al. (1997) were described as *traditional* and *constructivist*, whereas the one by Van Zoest et al. (1994) was referred to as *socio-constructivist orientation*. "This close match between the factors that emerged from the new survey and those that were identified using the surveys from which its items were derived is evidence of the validity of the survey used in the current study" (Beswick, 2005, p.46). The number of items according to Beswick (2005) was reduced due to the feedback that came from the participants in the pilot study. Therefore, 14 items were removed from the instrument to make it 26 items. Of the final 26 items, 24 were from the Van Zoest et al. (1994) survey and only 2, relating to the nature of mathematics, were from the Howard et al. (1997) survey.

The scoring of the 26-item beliefs survey was in such a way that the higher the score, the more the espoused beliefs' consistency with problem solving view of mathematics and corresponding views of mathematics teaching and learning. This was for all but the last 11 items, which were reversed because they were stated negatively (refer to appendix D). The survey also asked for personal data including gender, age, years of teaching experience, and teachers' highest qualifications. The instrument was not piloted in Malawi because it was already used in Australia hence it is a validated instrument to use.

3.4 Data Analysis: Cluster Analysis

The researcher used quantitative data analysis procedure using SPSS to conduct a cluster analysis (as described by Hair, Anderson, Tatham and Black, 1998) of the responses from the 65 teachers to the Likert scale questionnaire. Cluster analysis is a method which is used to classify individuals or objects that share characteristics that are different from another group. The actual number of clusters or groups was not known in the initial stage and the members for each group were not known either. In addition to that, cluster analysis was used to group the teachers depending on similarity in terms of their espoused beliefs. The particular cluster analysis procedure used in this study for the beliefs was hierarchical and agglomerative. That is, the respondents were initially considered to each comprise a separate cluster. Subsequent steps in the process combined the two most similar clusters from the previous step into a single cluster. In this study, Ward's method was employed. Specified to this case, the clustering process starts with all the 65 cases and then fuses the two which are least different to make 64 clusters, since most of them will still be clusters while they are still single cases. This then continues repeatedly until when the last two points are merged to make all the cases joined into a single cluster, see figure 3.1 on the next page.

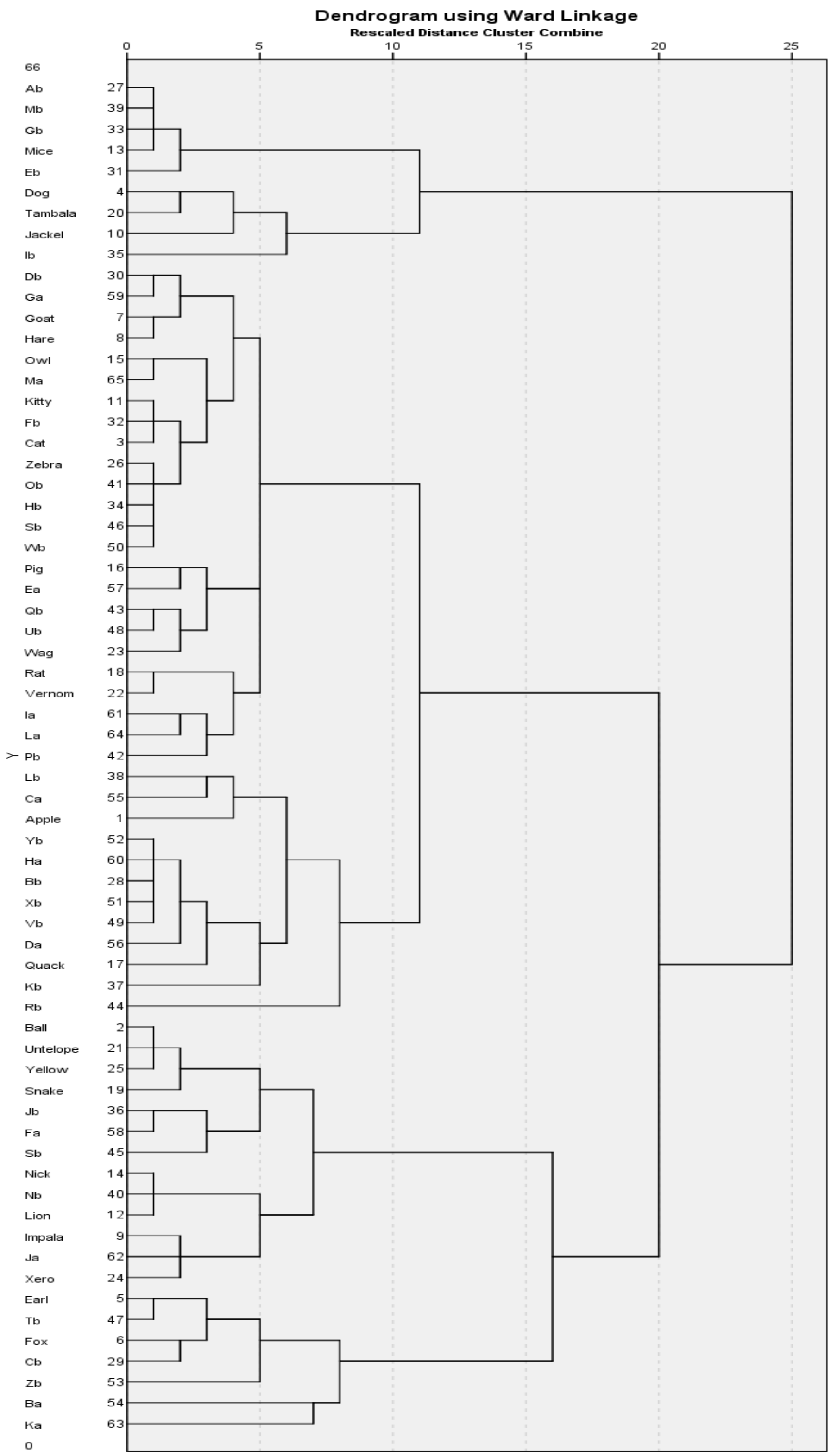

Figure 3.1: Summary of the Ward's method of clustering

Source : Researcher

The criterion for stopping the clustering is inspection of the agglomeration coefficients. That is, when the coefficients have a big difference, clustering can be stopped. For instance in figure 3.1, the last three clustering levels fall within the range of distance from eleven to twenty five. From this range, the level which had the highest number of clusters, four clusters stopping at three clusters, fell within a distance of eleven and sixteen; the level with three clusters that were stopping at two, fell within the distance of sixteen and twenty; and the level with two clusters had a range of distance from twenty to twenty to twenty five stopping at the point where there was only one cluster with all the members in one group. This shows that the level with three clusters and the other with one cluster had the highest and same difference of five unlike the level with two clusters which had a difference of four (refer with figure 3.1). The difference in the coefficients indicates that there is also a big difference between two members in the clusters, thus the need to separate these members by allocating them to two different clusters (Hair et al., 1998). Thus, the clustering can be considered to be good since it has partitioned the data into more similar objects within a group and differentiates groups with different members.

In this study, the researcher chose three clusters from the two with the highest difference between their coefficients because having one cluster will limit checking the differences between the members since the member of the cluster will be identified with similar characteristics only. On the contrary, three clusters will help see distinguishing factors that will enlighten the researcher on the factors that affect such kind of beliefs in the Malawian context. Coincidentally, the number of clusters used in this study has matched the ones used by the comparison study done by Beswick (2005). Hence, the comparison will be a lot easier than if the number of clusters did not match.

3.5 Reliability and Validity

3.5.1 Reliability

Factor analysis of Beswick's (2005) pilot study brought about two groups of the retained items consistent with Ernest's (1989b) instrumentalist and problem solving view of mathematics. An alpha reliability test for the two groups of items was conducted which resulted into alpha reliability coefficients of 0.78 for the latter and 0.77 for the former. For my study, to test for internal consistency of the items, I used SPSS to generate the alpha reliability test. From that, I found that the alpha reliability for the problem solving items was 0.76 and for the items consistency with instrumentalist view of nature of mathematics was 0.68. This therefore proves that the instrument was generating consistent information for my study with the comparison study by Beswick (2005).

3.5.2 Validity

As mentioned in the previous subsection, the groups of the items in the questionnaire were consistent with the instrumentalist and problem solving views of mathematics. This is so because the items in the questionnaire were capturing the conceptions that these views comprise as described in the literature as such the items were measuring the expected. With that, it is safe to say the instrument was valid.

3.6 Research ethics

Issues of ethics in this study were considered in the area of keeping anonymity and safeguarding participants' identity from being prone to tracking. The questionnaire (Appendix B) only requested for information about gender, age, professional qualification and number of years for teaching experience and participants were not required to indicate their names. Participants’ consent was also sought before the actual data collection. Thus those that took part in the study did at their own will and without being forced or persuaded in any form. Another measure that was taken was getting a letter of approval from Chancellor College (Appendix A) to the school’s head teachers and then the information about the study was told to the head teachers who explained to the teachers for their consent first. The teachers were guaranteed of their privacy, confidentiality, and also anonymity. Their participation in the study was voluntary and no information can be traced back to the individual participants as also identity of the schools where the teachers worked was anonymous.

3.7 Chapter Summary

This chapter has discussed the research design used in this study, the sampling technique, the questionnaire instrument used in this study, its reliability and validity, a discussion of the data analysis and finally how the issue of research ethics was taken into account in the study.

CHAPTER 4

Presentation and Interpretation of Data

4.0 Chapter Overview

This chapter will present and discuss the results from the analysis of the data generated. The discussion of the results is presented according to research questions. The last part of this chapter will give a summary of this chapter.

4.1 Presentation and discussion of results

During the discussion of results, knowledge of the beliefs about the nature of mathematics, beliefs about teaching and learning mathematics, were kept at the back of the researchers mind and were used as the basis for analysis. Categories of mathematics teachers' beliefs discussed in Table 2.1 were used as the framework for explaining results, and the information presented in the last row of the table are what correlated with notion of problem solving. That is, those teachers whose responses aligned with these were considered to fall under problem solving.

The teachers in this study responded to items 1 to 18 with extremely high and strong agreement in the favor of problem solving. All the items between items 1 to 18 are for problem solving, except for item 3 which is of Platonist view. A fairly good number of teachers (58% of the teachers) disagreed with telling children the answer as an efficient way of teaching mathematics (item 19). Although this was the case, 22% was undecided on this matter and 20% of the teachers agreed to imposition of answers to the learner. In addition to that, approximately half of the teachers (46%) were of the view that children ought to listen carefully to the teacher's explanation of mathematical concepts during lesson delivery while about one third (32%) disagreed to the assertion that learners must listen carefully for effective learning in mathematics to occur. On this same item (item 20), about one fifth of the teachers had a neutral stand about their role as instructors (22%).

4.1.1 Research question 1: Clusters in Teacher Beliefs Data

From the cluster analysis conducted to the data for the teachers' beliefs, three clusters emerged. The names of the clusters are flexible problem solvers, content understanding, and teaching for performance. The clusters were named following their characteristics drawn from their responses to the distinguishing items in Appendix D. The names were a slight diversion of the three beliefs of the nature, learning and teaching of mathematics because unlike in the theoretical framework, there were intersection of views. Flexible problems solvers portrays flexibility in accommodating learners' ideas, whether wrong or right and never advocated for lecture method of teaching mathematics. The content understanding cluster has teachers who are only concerned about imparting knowledge on learners. Content understanding teachers manifest partial flexibility which is biased towards fast or gifted learners. The teaching for performance cluster's teachers are those less likely to use learner-centred approaches and are very conscious of abiding by the curriculum's prescription and its strict attachment to time.

- Differences among the clusters are sorted in terms of what items vary significantly and each item's average total in each cluster is computed. Under Appendix D, there is a presentation of those items which has a significant variation in terms of their average total. Scores presented in this table can be distinguished by observing those that are approaching 1 as indicating a strong disagreement to the item while those close to 5 indicating a strong agreement.

Flexible problem solvers cluster has 20 teachers out of the 65 participating teachers. From the remaining 45 teachers, content understanding cluster has 36 teachers and teaching for performance cluster has 9 teachers. From the average total beliefs scores for each cluster in table 4.3 below, it is evident that flexible problem solvers cluster has the highest score of 96.0, seconded by content understanding with 95.5 and the lowest for teaching for performance, 84. This order will guide the detailed discussion of each cluster. The comparison study, the one done by Beswick (2005), computed a Pearson's correlation coefficient because the bio data in that study was continuous while for the current study, the data is categorical hence the Spearman's correlation coefficient is the appropriate one to use. For any Spearman's rho value less than half implies that there is no significant effect of the variable in focus. Evidently, all the variables in the bio data shown in Table 4.1 below registers a correlation coefficient of less than half which means that these variables, the teachers' gender, professional qualification and years of teaching experience has no significant effect on teachers' beliefs. For example, since the Spearman's rho of teachers' educational qualification and their teaching experience indicated in table 4.3 below is 0.120 and 0.433 respectively, then it entails that such characteristics have no effect on the teachers' choices of responses. This supports Cross (2009) cited Torff and Warburton (2005) who states that educational qualification and teaching experience does not affect what a person believes.

Table 4.1 Characteristics of Clusters

Cluster		Content Understanding	Flexible Problem solvers	Teaching for performance	Spearman's rho
Average total belief score		95.5	96.0	84	
teachers qualification	University Certificate of Education	0%	5%	0%	0.120
	Diploma	19%	25%	44%	
	Bachelor of Education Science	81%	70%	56%	
teaching experience	Less than 10 years	69%	80%	56%	0.433
	10 – 20 years	25%	15%	33%	
	More than 20 years	6%	5%	11%	
gender	Males	75%	55%	67%	0.242
	Females	25%	45%	33%	

4.1.1.1 Flexible Problem Solvers

Teachers in the flexible problem solvers cluster may be described as those whose view does not agree with teaching mathematics in an expository style of demonstrating, explaining and describing concepts and skills (Item 19). It is also not surprising that they strongly refused that mathematics is not computation (Item 20).

Contrary to the views of teachers in this cluster, this image of mathematics as being mainly about computations, is what Skemp (1978) characterized and likened to teaching for instrumental understanding. Probably these teachers viewed teaching mathematics as a process of negotiation since they also did not agree to imposing knowledge on learners by telling them the answer (Item 21). They did not mind if a student suggests a solution to a mathematical problem which they did not think of (Item 22). They also contended against lecture method by implication of Item 24. This item suggested that listening carefully to the teacher explaining is the most effective way of teaching. This cluster's teachers did not clearly state their stand on issues to do with indoctrination and coverage of concepts which is evidenced in their responses to items that they were neutral. These items (16 and 17) were concerning the issue of whether it was the teacher's responsibility or not to spoon feed learners with mathematical solutions clearly and concisely, and whether there is an established amount of mathematical knowledge to be covered at each grade level respectively. This may be due to the curriculum expectations discussed earlier and also due to the expectations from learners themselves that teachers should teach only that which they feel has a high probability of being examined in the national examinations (Kholowa et al., 2004). Since overall, teachers in this study believe that they have a responsibility to motivate learners and to teach them what they want, teachers may stand in a dilemma to teach against what they believe. On the other side of the coin, it may be that the teachers are undecided and that this may depend on the context in which this has to apply. Teachers in this cluster also hold the image that it is impossible for teachers to create a non-threatening learning environment of mathematics for all learners. According to Kholowa et al. (2004), large class size in Malawian secondary schools is one of the factors that affect quality of education in Malawi. That is to say, sometimes teachers teach in the way they teach with the knowledge of their class size in mind. In this context, teachers' propositions of the learning environment may be directed by their knowledge about the number of students they have. Definitely, it is hard for them to accommodate all the learners' needs in the environment they may create. Teachers' response to item 18 showed their concern about sequencing. This implies that, the teachers are constrained by the sequence provided by the syllabus and its prescribed textbooks. In addition to that, their response towards item 25 concerning coverage of all topics in the curriculum indicated that they believe it is important to cover all the topics without fail. Deductively, most of the teachers in this cluster agreed to ignore senseless ideas in pursuance of time at the expense of the student's opportunity (item 26) and not even bother to look for the source of the errors or misconceptions brought by the students (Item 23).

4.1.1.2 Content Understanding

The characteristics of this group are not very far from those of the previous group. This is evident in their average total belief score (95.5 versus 96.0), and also the dendrogram shown in figure 3.1 above and appendix E. The dendrogram shows that the next clustering stage would have joined the flexible problem solvers cluster and content understanding cluster to become one. This stage of clustering is occurring between the distance of 15 and 20 indicated in the x-axis. From the average total for each item the similarity is indicated in Items 18, 19, 20, 21, 24, 25 and 26. Teachers from the flexible problem solvers cluster had an average score of 1.8 while those from the content understanding a score of 2.4. In the same manner, the two clusters had the scores of 1.8 versus 2.4, 3.7 versus 4, 4.3 versus 4.5, 4.3 versus 4.3, 4.3 versus 4.4, 1.6 versus 2.2, and 1.3 versus 1.7 for items 19,20,21,24 and 25 respectively. If the power of rounding is induced on these decimals, then all of these will come to the same whole number except for the last item which will only differ in terms of strength of disagreement.

Otherwise both clusters are of the view that ignoring children's inappropriate ideas has a negative effect on children's learning. Flexible problem solvers' teachers view in this regard is weaker in the extent of agreement (which is 4 for agree) for avoiding student's ideas that seem senseless to the teacher than the strength of the teachers view in the content understanding cluster (which is 5 for strongly agree). This can be related to the Platonist view of mathematics where in the solution presentation and interpretation stage involves the teacher leaving the 'wrong' solutions untouched and focuses on the 'right' answers provided by learners. Teachers in these clusters would rather only work with students' ideas that make sense than waste time on what they may regard as useless. On the other hand, teachers in the content understanding cluster differ with the views of the flexible problem solvers cluster in the sense that they shared an idea that teachers can create a welcoming environment for all children (item 5) where the learners are given the opportunity to explore mathematical problems to find the way to the solution by themselves (item 16) without the teacher telling them the method (item 21).

In a framework which Van Zoerst et al. (1994) used, there were three types of the teaching and learning processes. The first one was content performance, the second was the content understanding and the last one was the learner interaction. In this framework, there were four stages to problem solving with each stage having the expected action for teacher and separately for students too. Van Zoerst et al.,'s (1994) names of the categories are not very different to the ones used in the clusters that emerged from the current study's data. Where Van Zoerst et al., (1994) has content performance, my study has content for performance; content understanding versus content understanding; and learner interaction versus flexible problem solvers.

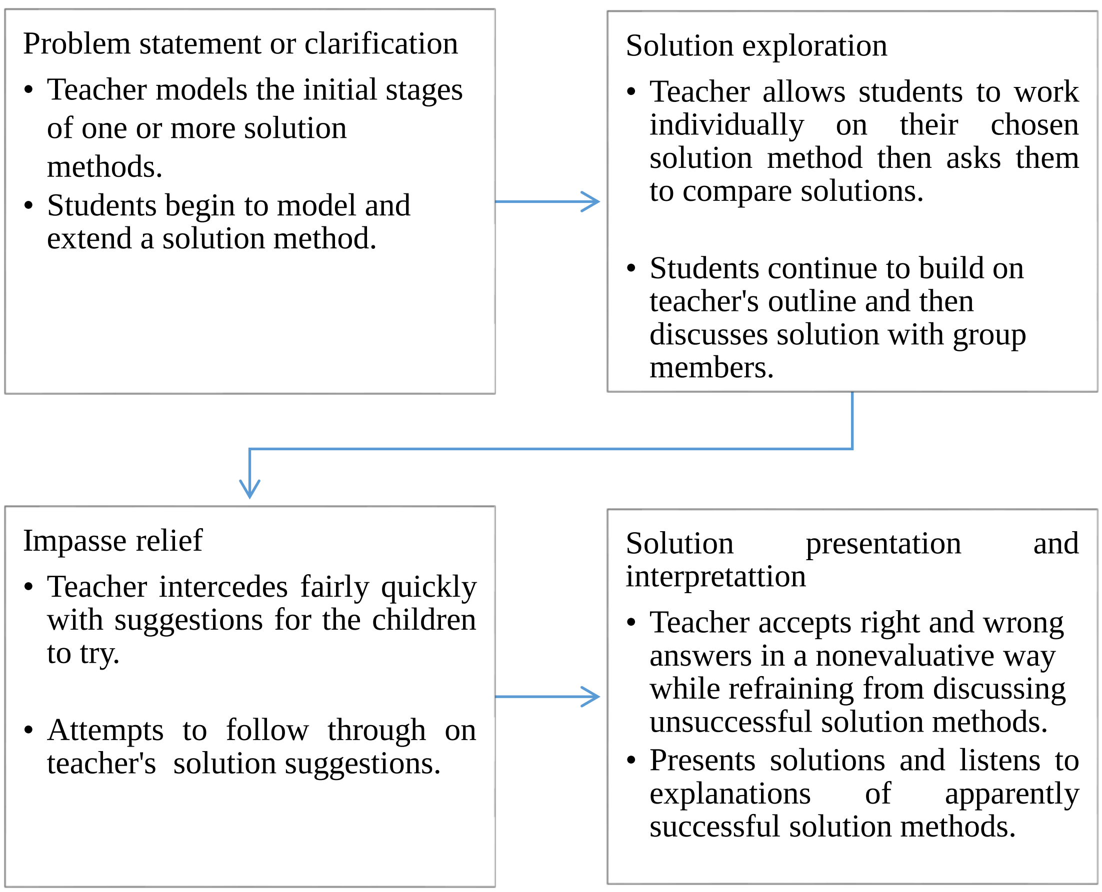

Figure 4.1 Content understanding view of teaching and learning process (Van Zoerst et al., 1994)

Source: Researcher

The content understanding cluster relates to the category of teachers described in the content understanding group modeled in figure 4.1 above. Teachers who were in the flexible problem solvers cluster were undecided on coverage of topics while the ones in the content understanding cluster were certain that there is an established or fixed amount of content to be covered at each grade level (as indicated by the average score for item 17 in appendix D). As pointed earlier in the literature review, Beswick et al. (2012) attributed to sticking to curricula materials without modification as falling under traditional ways of teaching which is problematic. This choice on coverage may explain why these teachers ignores ‘senseless’ responses from learners. Probably because they are working towards coverage of everything prescribed in the syllabus. Strict adherence to curricula demands is diverging from reform oriented initiatives (Schoenfeld, 2007) so is ignoring learners’ response on whatever grounds (Van Zoerst et al., 1994). This only shows that believing in the use of one sequence of topics is rigidity and is a hindrance for teachers to use learner centred approaches to teaching mathematics. No wonder, teachers in this cluster are not flexible to accommodate all learners’ ideas. They manifest rigidity to welcome learners’ suggestions that surpass their thinking and understanding or ideas which does not make sense. That is, they only are comfortable when the students contribute to the lesson with what was already on their plan or their imaginations and knowledge (item 22). A similar description is given in the model above, in the first stage of the problem exploration; the teacher puts the limits to the possible solutions so that students’ suggested solution paths should fall within such. Being of the view that ideas that do not make sense are worth disregarding, the teachers’ average score for the importance of understanding sources of students’ errors (item 23 in appendix D), 2.5, is approaching 3. This means that the teachers had no clear side on whether it is necessary or not. On the other hand, the score 2.5 may be interpreted as the teachers may not see this as necessary or may fall on the neutral ground previously stated.

4.1.1.3 Teaching for Performance

From the pattern of their responses, this cluster comprised teachers whose views were a slight deviation from the traditionalist. These teachers disagreed with most of the propositions which were stated in the direction of problem solving especially for the ones that distinguished the clusters. For instance, the average scores for all but four distinguishing items for teachers in this cluster were approaching 1 or 2, which implies disagreement. Exceptions from these items were one item (item 5) which registered one strong agreement and two distinguishing items (item 22 and item 23) got decimals approaching 3 towards problem solving as shown in table 4.2 above. Item 5 which these teachers agreed to was about a possibility that the teachers can create a learning environment where the learners cannot be threatened of which they registered a great possibility. Their response in this item is questionable as it depends on their perception of what it means by 'non-threatening'. According to Borgersen (1994), a non-threatening environment from his results is the one where students indulge in open ended questions that are contextual in groups. However, the teachers in this cluster, teaching for performance, have contrary views. They are bound by the curriculum, do not see the importance of understanding the sources of learners' errors, they see themselves as the custodians of knowledge and do not feel comfortable if a learner suggests a solution that they never thought of. Referring to the average scores for each cluster for items 25, 22, 23, in appendix D, a teacher in this cluster can fit in the category of teachers which Ernest

(1989a) states that have learning models where children must comply with instruction prepared with reference to the curriculum's prescription. Its best-fit nature of mathematics is the instrumentalist view (Ernest, 1989b). Unlike teachers in content understanding, teachers in this cluster view mathematics as primarily involving computation whereby the teacher imparts knowledge on learners as on empty slates. These teachers consider students as passive receivers of knowledge that only follow the teacher's guidance. This can be seen from a comparison of the scores for teachers' responses of the two clusters to items 19, 20, 21, and item 24 in table 4.2 as presented in the chart below.

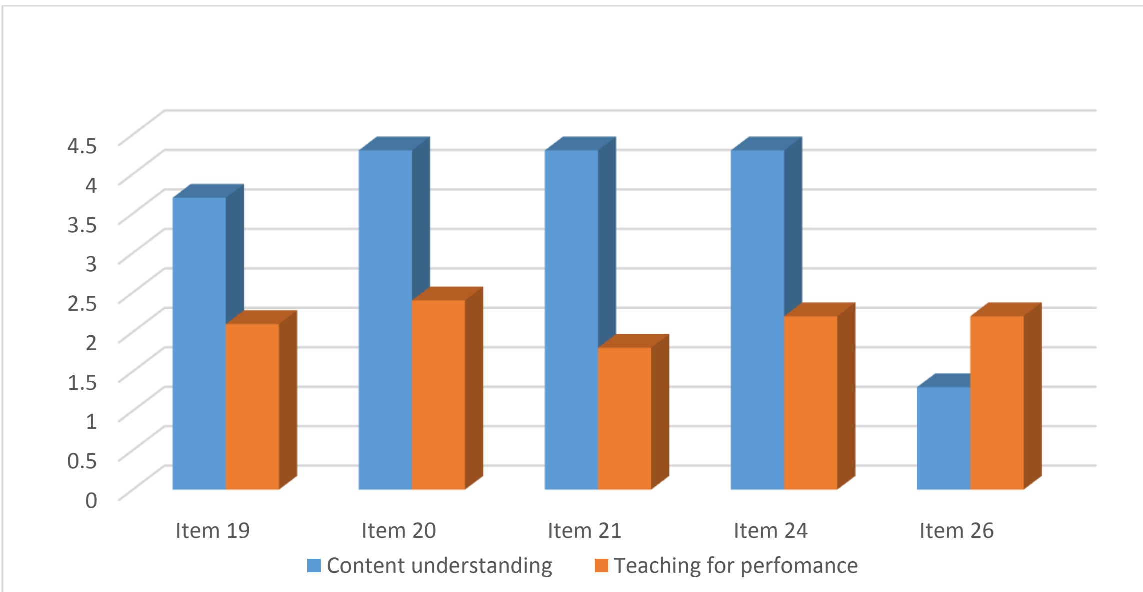

Figure 4.2 Item's average score comparison for Content understanding and Teaching for performance clusters

Source: Researcher

4.1.2 Research question 2: Comparison of Mathematics Teacher Beliefs from Beswick (2005) and my Study

Appendix F presents percentages of teachers' responses for each item in the questionnaire for this study and those as found by Beswick (2005). The results were converted to percentages for a meaningful comparison since the study by Beswick (2005) had 25 participants while my current study, the Malawi case, has 65 participants.

4.1.2.1 Similarities in the results from the two studies

Data generated from the two studies shows an overwhelming number of teachers agreeing with the first eleven items which all but item 4 are for problem solving view of mathematics (refer with appendix F). Item 4 portrays the Platonist view of mathematics. Such responses show that the teachers from both the current study and the one by Beswick in 2005 believes in giving learners autonomy. For instance, many from both sets of teachers agreed to the importance of giving learners the opportunity to direct their learning. According to Van Zoerst et al., (1994), teachers who believe in problem solving create a learning environment which allows learners to decide what they want to learn and how they want to learn it. Such teachers do not put limits to the learners thinking (Van Zoerst et al., 1994). This is portrayed in the great percentages of teachers from both studies who find it necessary to allow learners to reflect and evaluate their own understanding. The reflection, according to Van Zoerst et al., (1994), may be done during solution exploration stage or the impasse relief stage of problem solving to a mathematical problem. Such belief accommodates learners' mistakes and corrections which they do in the process just like teachers from both the Malawian and Australian studies agreed in great numbers to the use of collaborative ways of teaching mathematics where learners learn from their friends and are able to discuss their ideas in groups. These results correlate with the belief of mathematics as a socially constructed knowledge as discussed by Yackel et al., (1991), Cobb et al., (1992), Borgersen 1994, Handal 2003, and Webb and Webb, (2004).

4.1.2.2 The differences between results from the Malawi and Australia studies

In agreement to Levin's (2015) assertion that the contextual nature of beliefs is manifested in comparative studies for two different contexts, the comparison in this study registered some differences discussed below.

As has been stated earlier, items 16 to 26 were either Platonist or Instrumentalist. Unlike teachers from Beswick's (2005) study, a large number of the participants in the current study agreed to those especially from 17 to 26 (see the bar graph below in figure 4.3). These items are related to curricula demands and this connects with argument by Tschannen-Moran, Salloum, and Goddard (2015) who stated that a country's curriculum may act as a constraint to how teachers present knowledge. Similar findings were registered by Schoenfeld (2007) were the U.S curriculum has an effect on teachers ways of teaching mathematics.

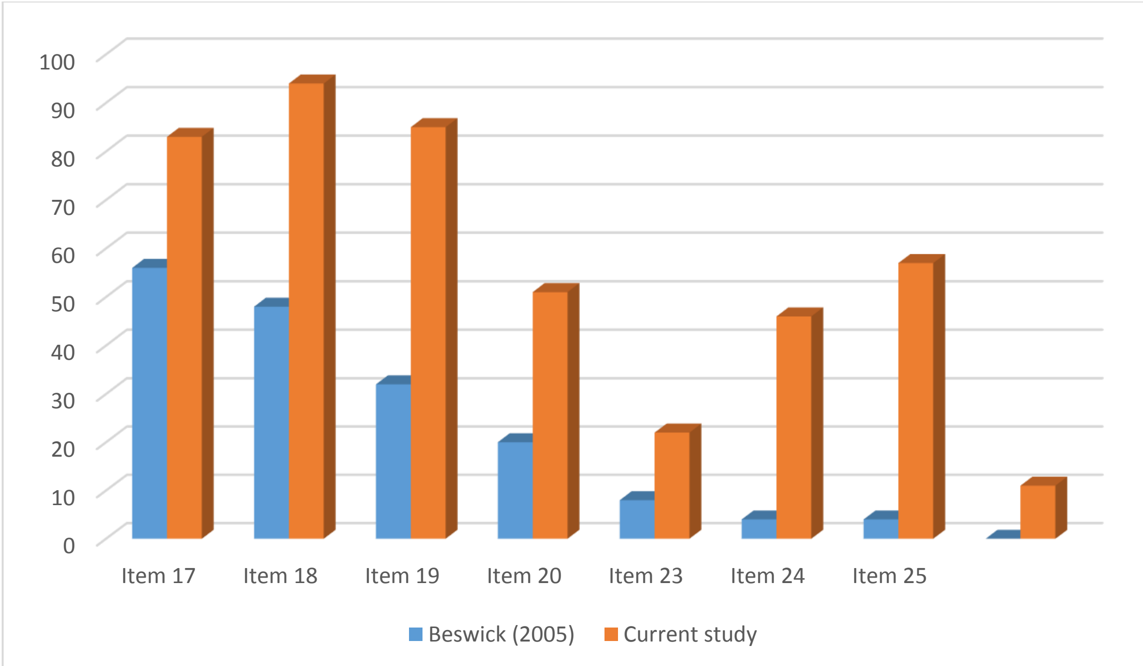

Figure 4.3 Differences between teachers’ responses for Current and Beswick's (2005) studies

Source: Researcher

If closely looked at, teachers’ responses to item 16 and item 21 may seem contradicting but on the other hand, explanatory without contradictions. According to Cross (2009), it is possible for teachers to hold two opposite beliefs without interference to one another. The stand of contradiction will be disregarded in this case from an argument made by Leatham (2006) discussed earlier about sensible systems. Explanation which these responses may offer is that many teachers in the Malawian context believe that they have a responsibility to provide learners with ‘clear and concise solutions’ (item 16), although they do not agree to do this in a spoon feeding manner. Responses to these items in the Malawian context match with those found in the Australian context by Beswick (2005) but for different reasons. Beswick (2005) attached teachers’ responses to these items to beliefs which the teachers hold centrally as opposed to peripheral beliefs, while in the Malawian context, this can be connected to the trend in their responses which manifests teachers' restriction by the curriculum. That is, although the teachers would not advocate for spoon-feeding, there is still an extent to which they use telling due to the demands pressed by the over-burdened syllabus. For item 20, more than half of the teachers agreed that mathematics is computation compared to one fifth of the Australian teachers. It is a possibility that Malawian teacher are of this view to cover a considerable topics just to prepare their learners for public examination to increase the chance of the learners to pass public examination just like it is the case with Satterly's (1989) argument. Furthermore, about one third of the Malawian teachers were undecided about the proposition laid by item 20 against one fifth of Australian teachers and many of the Australian teachers disagreed to this while few (17%) of the Malawian teachers disagreed. In addition to that, most of the teachers for the Australian context did not agree to stick to one correct sequence of topics and also fewer than half of the teachers in the Australian context agreed to the importance of having one correct sequence provided by a textbook. A considerable number of teachers in the Australia group were undecided on this matter and very few disagreed. Although overwhelming numbers of teachers for both countries were of the view that it is important that the teacher allows their learners to evaluate their own mathematical thinking and look back on their own, 85% of the Malawian teachers believe in teacher centred method of demonstrating in a way of exposing mathematical knowledge as being effective (Item 19). The remaining 15% of teachers split to 4% of the undecided and 11% for those that disagreed. On the other hand, the Australian teachers had only 32% for demonstration fully done by the teacher, 40% for those with a neutral stand and 28% for those that disagreed. Teacher responses to Item 24 also spread the teachers from the two studies differently. For

Beswick's (2005) study, only a 4% of the teachers agreed to lecture method portrayed in Item 24, 36% neither agreed nor disagreed and the rest against lecture method. On the other hand, my study indicates that close to a half (46%) of the teachers agreed to the lecture method suggesting that they were for imposition of mathematical knowledge, 22% was neutral and about one third (32%) was against. This shows that most teachers in the Australian study were not for lecture method of teaching while most of the Malawi teachers were. Interestingly, the teachers' level of agreement to problem solving started to decline for items related to the scope of mathematics as prescribed by the curriculum. Extremely high numbers of teachers seemed to be constrained by the curriculum. As such, instead of considering giving children autonomy for their learning, their choice of what is to be taught and how it is sequenced are driven by the curriculum rather than children's needs. Ninety four percent of the teachers agreed that there is a correct sequence of which all teachers must follow in presenting mathematical content. In addition to that, eighty three percent of the teachers agreed that there is a set amount of mathematical content that should be covered at each grade level. With the high percentage of teachers agreeing to that, one can say that most teacher believe that no matter what, the teacher ought to cover everything that is in the curriculum without fail. From the way teachers responded to the items discussed above, one can draw from it that the teachers believe that children have to listen carefully to the teacher to be told what the expected knowledge is by the curriculum for the children to acquire. This is portrayed in the way the teachers responded to the last seven items.

In this case, in the Malawian context, these teachers also have this pressure to cover everything so that their learners can stand a chance of passing national examinations well.

4.2 Chapter Summary

This chapter has presented a discussion of the results found from the analysis in terms of the research objectives. The next chapter is the conclusion and implications of the study.

CHAPTER 5

Conclusions and Implications

5.0 Chapter Overview

This chapter has four sections; the conclusion for the study, the implications of the findings, limitations of the study, suggestions for future studies and finally, a summary of the chapter. Under the conclusion section, the study and its findings are summarized with reference to the objectives and the theoretical framework. The implications section indicates what the findings mean in the Malawian society and what issues the study may raise. Finally, suggestions for future studies indicate other studies which may build on the findings from this study.

5.1 Conclusion of the Study

5.1.1 Teacher beliefs about the nature of mathematics, its teaching and learning

Contrary to Gaskey (1986) this study has found that teachers' experience in teaching has no effect on their beliefs. This study also refutes Van Zoerst et al.,'s (1994) and Beck and Buehl's (2005) findings that professional development determines the kind of beliefs one will hold. This study has found that there is no relationship between belief and professional development. In addition to that, it is evident in this study that teachers' beliefs about the nature of mathematics are problem solving and its corresponding teaching and learning beliefs. This is true for all circumstances except when their beliefs are challenged by the curriculum.

5.1.1.1 Problem solving nature of mathematics and its corresponding beliefs about teaching and learning Mathematics

Teachers' belief about mathematics as problem solving is revealed from their responses which showed the view that mathematics is a flexible subject which accommodates teaching strategies that put the learner at the centre, and the teacher's expectations of the students' actions in classroom related to Van Zoest et al., (1994)'s social interaction group. A majority of the teachers were of the view that mathematics discovery is a continuous and an ever-changing subject which is not only about getting the solution but is also concerned about the process of getting to the solution which is adding to Lakatos' (1976) contribution to how mathematics develops. Following that, teachers in this study believed that it is important that teachers create opportunities for learners that will give them autonomy to their own learning just as Borgersen (1994) discovered that involving learners in open ended discovery of geometric concepts is vital to teaching the concepts. Such ways of giving learners autonomy is by using group work where learners can justify their solutions to mathematical problems to fellow learners and discuss. Furthermore, their responses portrayed the belief that raising learners' curiosity and motivating the learners to enjoy learning mathematics by eliminating all threatening factors in the classroom environment may be profitable for effective teaching and learning. Considering that learners may encounter obstacles during learning, teachers responses indicated that ignoring the insensible solutions given by learners may be such threatening factors and that may hinder learning. Contending against spoon-feeding methods, teachers in this study considered such obstacles and tension as part of learning which has their own significance in the process if they give room to understand the sources of the learners' errors. On the other hand, teachers in this study showed a belief that learners are capable to discover new solution paths or concepts which the teacher or other mathematicians never discovered.

5.1.1.2 Platonist nature of mathematics and its corresponding teaching and learning

As stated earlier, the teachers' belief about problem falls short when it faces the curriculum as an obstacle. Teachers studied in this research were very concerned about coverage of all the topics prescribed in the curriculum and feel obliged to cover everything in the syllabus (Sartterly, 1989). They also showed a deep concern for the sequence in which they cover the topics. Thus, they showed belief that there is one proper way of arranging mathematics concepts when teaching. This kind of belief aligns with what Ernest (1989b) call Platonist view of mathematics where the concepts have a structure that was there and will always be there. From Ernest's (1989b) description of Platonism, one may regard these teachers that consider mathematical concepts as related in a kind of structure to fall under this group. That is, the teachers' strict adherence to the prescription by the curriculum shows that they consider mathematics to have a linear structure which cannot be distorted or modified as far as relationship among concepts is concerned. These issues of coverage and sequencing are quite explanatory of why the teachers viewed teachers as explainers and learners as passive listeners which connects with Van Zoerst et al.,'s (1994) corresponding teaching strategies. Certainly to cover a lot in the right order, the learners have to take their role as knowledge absorbers hence some teachers in this study opt for expository method of teaching mathematics. There is a very thin line between the Platonist and instrumentalist. With that, the only point where the teachers showed to have an instrumentalist view of mathematics in this study is where mathematics was considered to be mainly computation. This belief can also be a result of restrictions posed by the curriculum.

5.1.2 How similar or different are the results found in this study compared to the findings by Beswick (2005) in Australia

5.1.2.1 Similarities

The main similarity that can be noted in these two studies is that, clusters found in both studies cannot be exclusively categorized as Beswick's (2005) framework for studying beliefs (Instrumentalist, Platonist and Problem solving with their corresponding teaching and learning views). That is, the clusters found had overwraps of views depending on the contexts. Although this is their similarity, the clusters found in the Australian study and this Malawian study were not characterised by similar aspects and the percentage of members in the clusters were not the same for the two settings.

Another similarity from the results of the two contexts were similar in areas to do with motivation of learners, enjoyment of the subject, and making the learners part of the process of solution discovery. Teachers from both contexts showed acceptance of accommodating both gifted and slow learners in the learning process and creating a learning environment that is non-threatening for the learners. The use of collaborative methods like pair work and group work was approved by teachers in both studies although teachers from both contexts held the view that they have a responsibility to provide the procedure and the final solution clearly to the learners. Although the teachers agree in the different ways discussed above, they have other issues where they do not. The next subsection gives a brief summary of the differences.

5.1.2.2 Differences

Unlike the Malawi case which registered restrictions by the curriculum for teachers to believe in problem solving, the Australian case had no barriers in that regard. This is because in Malawi the curriculum of mathematics is implemented in a manner that prepares learners for examinations unlike the one for Australia which is outcome based. While Malawian teachers were more concerned about coverage of concepts, most Australian teachers were not mindful of that. A complete divergence in the responses of the Malawian teachers and Australian teachers was the issue of the nature which translated to procedural fluency for learners. That is, Australian teachers were against considering mathematics as computation unlike their counterparts. Most Malawian teachers also held to the lecture and telling way of teaching as compared to the Australian teachers. Probably, this was favourable in the interest of time expected of them to cover the syllabus which they regarded as crucial to complete all topics. Such differences compliment the argument that beliefs are contextual by Levin (2015).

5.2 Implications

The following are the implications of the study's findings;

> Firstly, one can get the impression that the two curriculums in the two contexts of Malawi and Australia may be different in the way they are delivered to the teachers and what they expect from the teachers too.

This study shows that there is a degree to which the teachers have embraced reform initiatives and has a small distance to reach to full appreciation of problem solving strategies. Supplementary to this point is that the Malawi government's effort of training in service teachers seems not to have been wasted. Malawi institute of education's report confirmed the urge to move from the traditional way of teaching to a more child centred approach (Malawi Institute of Education, 1990).The Malawian mathematics current curriculum (Ministry of Education, 2015) continues the trend and stipulates that teachers should be able to create opportunities for learners to be able to manipulate data and which will help them to use mathematics to solve mathematics related practical problems in their everyday life. The teachers should also encourage and motivate creativity and innovation and problem solving. From the results found in this study, it shows that teachers' perceptions are changed towards motivating learners, accommodating learners' autonomy and creativity.

In addition to that, the findings for this study add value to the literature that argues that beliefs are contextual. The curriculum has a major constraining power over teachers' beliefs in the Malawian context as such there is a proper need for an awareness to teachers that the curriculum should not replace teachers creativity and problem solving innovations.

5.3 Limitations of the study

Just like any other study, this study was faced with a number of limitations.

The first limitation that this study encountered is the one associated with low number of participants. This problem was a consequence of the target population which was mathematics teachers only. Regardless the fact that most schools have few mathematics teachers, the number of the teachers found at a school was very low than expected. For instance, the highest number of mathematics teachers in targeted schools was eight and this was only one case, the rest were very low. The expectation was to get at least 3 mathematics teachers per school but some school could only have one mathematics teacher for all forms.

Furthermore, some teachers in sample schools were not willing to participate and the researcher could not force them due to ethical reasons. Although this is a limitation in terms of the low number of teachers that participated, it is strength that ethical guidelines were observed and teachers were given the chance to exercise their right to decline. Another limitation is that the results of this study cannot be generalized to all mathematics teachers in Malawi because the sample was not representative to the total population since the study was conducted in two districts only. However, findings from this study contribute to the existing literature about mathematics teachers' belief, especially for mathematics education in Malawi.

One more limitation of the study is that the relevant literature that I found was old. As such, I made sure that the recent literature that I found was used appropriately. Although this is the case, the old literature that I used was relevant and contributed well to the study. This study will therefore enrich the deficient literature for beliefs about the nature, teaching and learning of mathematics.

5.4 Suggestions for future studies

Since this study's main focus was teachers' professed beliefs, other studies may build on this and study teachers' enacted beliefs and see if there is a relationship between the two. Such studies may use interviews and observation of mathematics lessons to gain a deeper insight into the teachers' beliefs and include teachers' justifications of their actions.

It has been observed in this study that the Malawian curriculum is a factor in teachers' beliefs. As such, future studies may study the curriculum and its features and also seek teachers' views of how the curriculum affects their ways of teaching in the modern way.

Other studies may also study teachers' beliefs from their learners' perspective. This may be done to see also the learners' views of the nature of mathematics, its teaching and learning as a factor of the experiences that their teachers expose them to. Such studies will enrich the literature.

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APPENDICES

Appendix A: Letter of introduction from Chancellor College

UNIVERSITY OF MALAWI

PRINCIPAL
Richard Tambulasi, B.A (Pub Admin), BPA (Hons), MPA, Ph.D

CHANCELLOR COLLEGE
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OFFICE OF THE DEAN OF EDUCATION

17 May 2016

TO WHOM IT MAY CONCERN

Dear Madam/Sir,

RE: MRS ESTHER SHAWA

This is to confirm that Mrs Esther Shawa is a Masters student under Faculty of Education at Chancellor College, University of Malawi. Mrs Shawa is required to conduct Mathematics Education research in Malawi secondary schools as part of her Masters research study. Please assist her accordingly.

If you have any questions or queries about Mrs Shawa or her research study, please feel free to contact me or the Dean of Education.

Your assistance will be greatly appreciated

Dr Mercy Kazima Kishindo
Coordinator of Mathematics Education Masters programme
Email: mkazima@cc.ac.mw,
Phone: 0111955757 (office)

Appendix B: Questionnaire used for data collection

Beliefs survey

This survey has a focus on teachers' beliefs about mathematics and mathematics teaching. As a teacher educator, we want you to answer these questions in a way that reflects what you think is important for the teachers you educate.

In the first 26 items, please mark one point along the scale (from strongly disagree to strongly agree) for each statement. In the next five questions (27–31), we ask you to write your response in full. Finally, select one alternative in question 32.

	Strongly disagree	Disagree	Neither agree nor disagree	Agree	Strongly agree
1. A vital task for the teacher is motivating children to solve their own mathematical problems.	1	2	3	4	5
2. Ignoring the mathematical ideas that children generate themselves can seriously limit their learning.	1	2	3	4	5
3. It is important for children to be given opportunities to reflect on and evaluate their own mathematical understanding.	1	2	3	4	5
4. It is important for teachers to understand the structured way in which mathematics concepts and skills relate to each other.	1	2	3	4	5
5. Effective mathematics teachers enjoy learning and 'doing' mathematics themselves.	1	2	3	4	5
6. Knowing how to solve a mathematics problem is as important as getting the correct solution.	1	2	3	4	5
7. Teachers of mathematics should be fascinated with how children think and intrigued by alternative ideas.	1	2	3	4	5

8. Providing children with interesting problems to investigate in small groups is an effective way to teach mathematics.	1	2	3	4	5
9. Mathematics is a beautiful, creative and useful human endeavour that is both a way of knowing and a way of thinking.	1	2	3	4	5
10. Allowing a child to struggle with a mathematical problem, even a little tension, can be necessary for learning to occur.	1	2	3	4	5
11. Children always benefit by discussing their solutions to mathematical problems with each other.	1	2	3	4	5
12. Persistent questioning has a significant effect on children's mathematical learning.	1	2	3	4	5
13. Justifying the mathematical statements that a person makes is an extremely important part of mathematics.	1	2	3	4	5
14. As a result of my experience in mathematics classes, I have developed an attitude of inquiry.	1	2	3	4	5
15. Teachers can create, for all children, a non-threatening environment for learning mathematics.	1	2	3	4	5
16. It is the teacher's responsibility to provide children with clear and concise solution methods for mathematical problems.	1	2	3	4	5
17. There is an established amount of mathematical content that should be covered at each grade level.	1	2	3	4	5
18. It is important that mathematics content be presented to children in the correct sequence.	1	2	3	4	5

19. Mathematical material is best presented in an expository style: demonstrating, explaining and describing concepts and skills.	1	2	3	4	5
20. Mathematics is computation.	1	2	3	4	5
21. Telling the children the answer is an efficient way of facilitating their mathematics learning.	1	2	3	4	5
22. I would feel uncomfortable if a child suggested a solution to a mathematical problem that I hadn't thought of previously.	1	2	3	4	5
23. It is not necessary for teachers to understand the source of children's errors; follow-up instruction will correct their difficulties.	1	2	3	4	5
24. Listening carefully to the teacher explain a mathematics lesson is the most effective way to learn mathematics.	1	2	3	4	5
25. It is important to cover all the topics in the mathematics curriculum in the textbook sequence.	1	2	3	4	5
26. If a child's explanation of a mathematical solution doesn't make sense to the teacher it is best to ignore it.	1	2	3	4	5

Appendix C: Percentages of responses arranged in ascending order

	Strongly agree and agree (%)	Neither agree nor disagree (%)	Strongly disag and disagree (%)
1. Mathematics is a beautiful, creative and useful human endeavour that is both a way of knowing and a way of thinking.	96	1	3
2. It is important for children to be given opportunities to reflect on and evaluate their own mathematical understanding	95	3	2
3. It is important for teachers to understand the structured way in which mathematics concepts and skills relate to each other.	95	2	3
4. Children always benefit by discussing their solutions to mathematical problems with each other.	94	3	3
5. Teachers can create, for all children, a non-threatening environment for learning mathematics.	92	5	3
6. Providing children with interesting problems to investigate in small groups is an effective way to teach mathematics.	91	5	4

7. Knowing how to solve a mathematics problem is as important as getting the correct solution.	88	8	4
8. Justifying the mathematical statements that a person makes is an extremely important part of mathematics.	88	6	6
9. Ignoring the mathematical ideas that children generate themselves can seriously limit their learning.	88	3	9
10. Persistent questioning has a significant effect on children's mathematical learning.	85	14	1
11. A vital task for the teacher is motivating children to solve their own mathematical problems.	85	3	12
12. Teachers of mathematics should be fascinated with how children think and intrigued by alternative ideas.	83	14	3
13. As a result of my experience in mathematics classes, I have developed an attitude of inquiry.	83	9	8
14. It is ok for me if a child suggested a solution to a mathematical problem that I hadn't thought of previously.	80	6	14
15. If a child's explanation of a mathematical solution doesn't make sense to the teacher it is not right to ignore it.	78	11	11
16. Allowing a child to struggle with a mathematical problem, even a little tension, can be necessary for learning to occur.	75	11	14

17. It is necessary for teachers to understand the source of learners' errors; follow-up instruction will correct their difficulties.	75	3	22
18. Effective mathematics teachers enjoy learning and 'doing' mathematics themselves.	70	5	25
19. An effective mathematics teacher, who is a facilitator of mathematics learning, does not tell the learners the answer.	58	22	20
20. Listening carefully to the teacher explain a mathematics lesson is not an effective way to learn mathematics.	32	22	46
21. It should not be a priority to cover all the topics in the mathematics curriculum in the textbook sequence over understanding of the topics by learners.	26	17	57
22. Mathematics is not computation.	17	32	51
23. It is the teacher's responsibility to let learners be accountable for their learning so that they explore possible solution paths which lead them to the solution and not providing for them	17	9	74
24. Mathematical material is best presented in an exploratory style: with learners investigating open ended real world problems in groups, in a process of conjecturing and refutations.	11	5	84

25. Mathematics is a broad subject which every bit of its content may or may not be covered as prescribed by the curriculum	8	9	83
26. There is no one correct sequence for teaching mathematics since all mathematics are related hence learners needs should direct the sequencing of the topics to be taught.	3	3	94

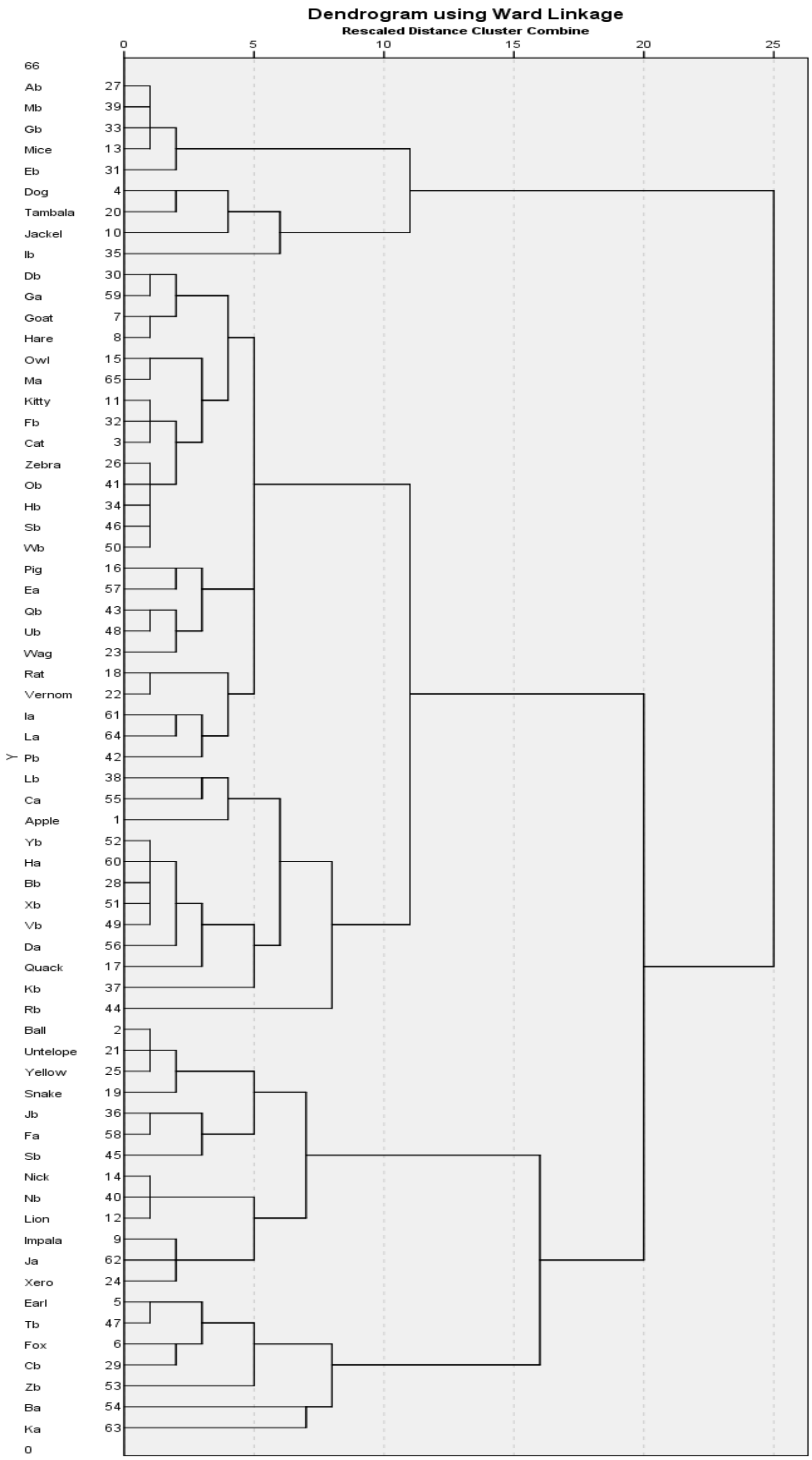
Appendix D: Items for distinguishing clusters and their average total score

Item number in Table 4.1	**Beliefs survey item**	**Flexible problem solvers**	**Content understanding**	**Teaching for performance**
5	Teachers can create, for all children, a non-threatening environment for learning mathematics.	2	4.4	4.6
16	It is NOT the teacher's responsibility to provide children with clear and concise solution methods for mathematical problems.	3	2.4	2.4
17	There is NOT an established amount of mathematical content that should be covered at each grade level.	2.7	1.8	2.1
18	It is NOT important that mathematics content be presented to children in the correct sequence.	2.4	1.8	1.8
19	Mathematical material is NOT best presented in an expository style:	4	3.7	2.1

	demonstrating, explaining and describing concepts and skills.			
20	Mathematics is NOT computation.	4.5	4.3	2.4
21	Telling the children the answer is NOT an efficient way of facilitating their mathematics learning.	4.3	4.3	1.8
22	I would NOT feel uncomfortable if a child suggested a solution to a mathematical problem that I hadn't thought of previously.	3.6	2.3	3
23	It is NECESSARY for teachers to understand the source of children's errors; follow-up instruction will correct their difficulties.	2.4	2.5	2.9
24	Listening carefully to the teacher explain a mathematics lesson is NOT the most effective way to learn mathematics.	4.4	4.3	2.2
25	It is NOT important to cover all the topics in the mathematics	2.2	1.6	2.2

	curriculum in the textbook sequence.			
26	If a child's explanation of a mathematical solution doesn't make sense to the teacher it is NOT best to ignore it.	1.7	1.3	2.2

Appendix E: Summary of the Ward’s method of clustering



Appendix F: A Comparison of Teacher Beliefs Survey Responses in Percentages for my Study and those by Beswick (2005)

Number of teachers (%)	Strongly agree And agree		Neither agree nor disagree		Strongly disagree and disagree	
	Current study	Beswick (2005)	Current study	Beswick (2005)	Current study	Beswick (2005)
1. A vital task for the teacher is motivating children to solve their own mathematical problems.	85	100	3	0	12	0
2. Ignoring the mathematical ideas that children generate themselves can seriously limit their learning.	88	96	3	4	9	0
3. It is important for children to be given opportunities to reflect on and evaluate their own mathematical understanding.	95	92	3	8	2	0
4. It is important for teachers to understand the structured way in which	95	92	2	4	3	4

mathematics concepts and skills relate to each other.						
5. Effective mathematics teachers enjoy learning and 'doing' mathematics themselves.	70	88	5	8	25	4
6. Knowing how to solve a mathematics problem is as important as getting the correct solution.	87	88	8	4	5	8
7. Teachers of mathematics should be fascinated with how children think and intrigued by alternative ideas.	83	88	14	4	3	8
8. Providing children with interesting problems to investigate in small groups is an effective way to teach mathematics.	91	88	5	0	4	12
9. Mathematics is a beautiful, creative and useful human endeavour that is both a way of knowing and a way of thinking.	96	76	11	12	3	12
10. Allowing a child to struggle with	75	72	11	24	14	4

a mathematical problem, even a little tension, can be necessary for learning to occur.						
11. Children always benefit by discussing their solutions to mathematical problems with each other.	94	72	3	20	3	8
12. Persistent questioning has a significant effect on children's mathematical learning.	85	68	14	28	1	4
13. Justifying the mathematical statements that a person makes is an extremely important part of mathematics.	88	64	6	36	6	0
14. As a result of my experience in mathematics classes, I have developed an attitude of inquiry.	83	64	9	24	8	12
15. Teachers can create, for all children, a non-threatening environment for learning mathematics.	92	64	5	20	3	16
16. It is the teacher's responsibility to provide children with clear and	74	60	9	12	17	28

concise solution methods for mathematical problems. [1]						
17. There is an established amount of mathematical content that should be covered at each grade level. [1]	83	56	9	28	8	16
18. It is important that mathematics content be presented to children in the correct sequence.[1]	94	48	3	32	3	20
19. Mathematical material is best presented in an expository style: demonstrating, explaining and describing concepts and skills. [1]	85	32	4	40	11	28
20. Mathematics is computation	51	20	32	20	17	60
21. Telling the children the answer is an efficient way of facilitating their mathematics learning. [1]	20	16	22	12	58	72
22. I would feel uncomfortable if a child	14	12	6	4	80	84

[1]*These items were reversed so that the actual picture of teachers' views towards the nature, teaching and learning of Mathematics istowards problem solving*

suggested a solution to a mathematical problem that I hadn't thought of previously. [1]						
23. It is not necessary for teachers to understand the source of children's errors; follow-up instruction will correct their difficulties. [1]	22	8	3	8	75	84
24. Listening carefully to the teacher explain a mathematics lesson is the most effective way to learn mathematics. [1]	46	4	22	36	32	60
25. It is important to cover all the topics in the mathematics curriculum in the textbook sequence. [1]	57	4	17	4	26	92
26. If a child's explanation of a mathematical solution doesn't make sense to the teacher it is best to ignore it. [1]	11	0	11	0	78	100